Chap1 Kinematics

Motion in One Dimension

Displacement

vector

- $\triangle x = x_f - x_i$
- SI unit: m
- displacement

scalar

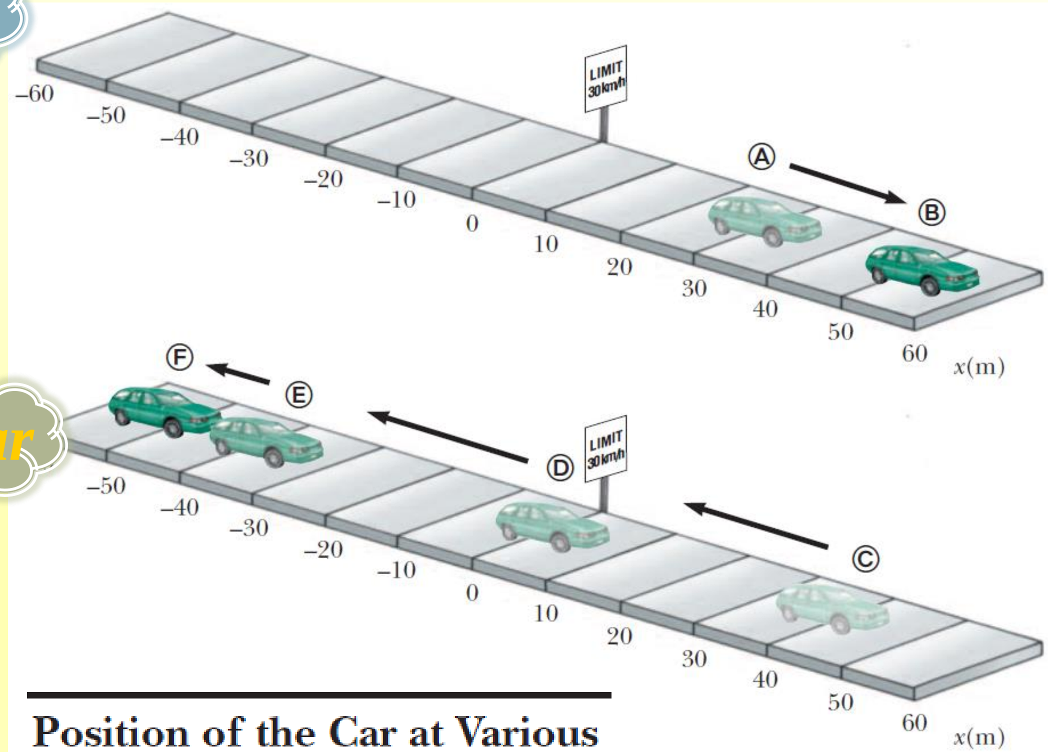
vs distance

Ⓐ→Ⓑ: $\triangle x = 52\ m - 30\ m = +22\ m$

Ⓒ→Ⓕ: $\triangle x = -53\ m - 38\ m = -91\ m$

Ⓐ→Ⓓ: $\triangle x = 0\ m - 30\ m = -30\ m$

$d = 22\ m + 52\ m = 74\ m$

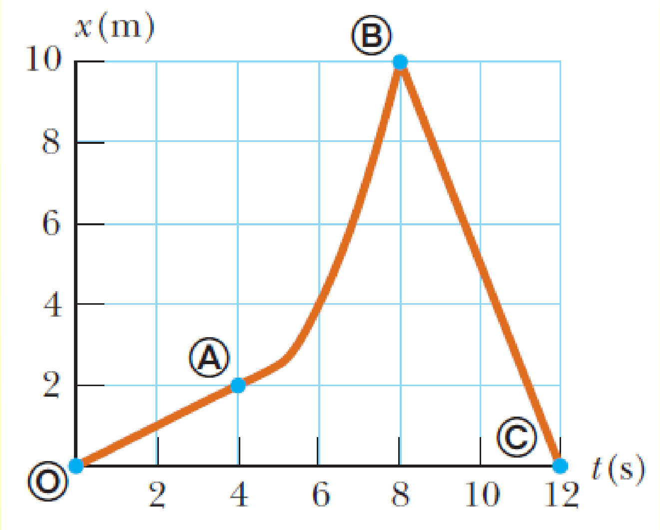


Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	− 37
Ⓕ	50	− 53

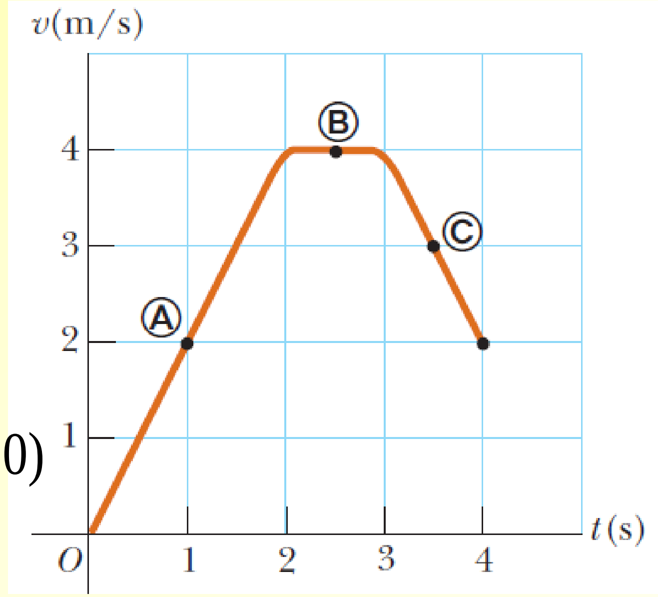
Velocity (SI unit: m/s)

- **average velocity:** $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x - x_0}{t - 0}$
- **instantaneous velocity:** $v = \dfrac{\Delta x}{\Delta t} \quad (\Delta t \to 0)$
- **average speed:** $\bar{v} = \dfrac{d}{t}$
- **instantaneous speed:** ***Magnitude*** **of the instantaneous velocity**

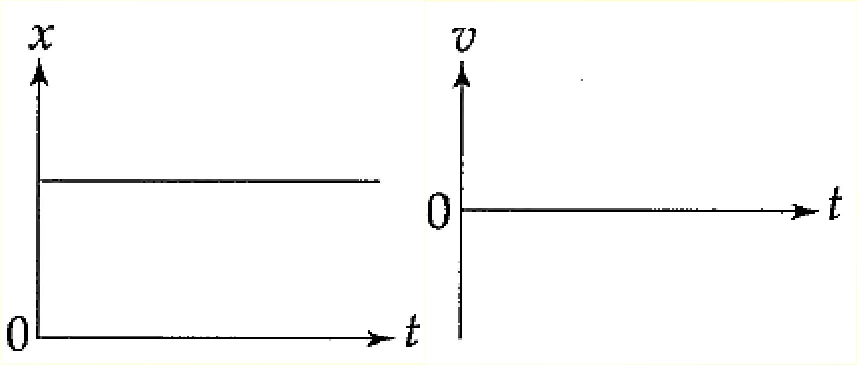


Acceleration (SI unit: m/s^2)

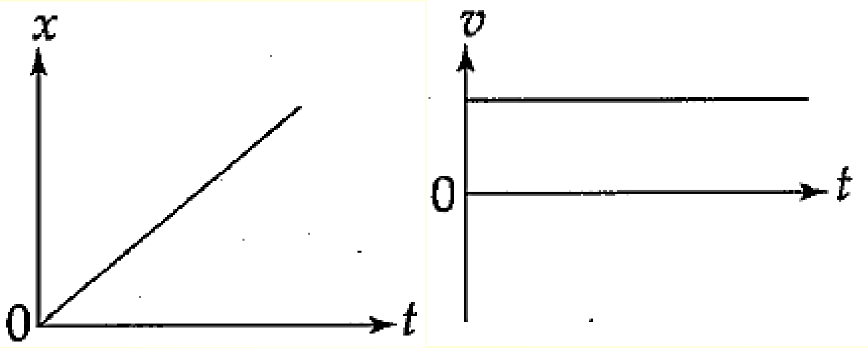
- **average acceleration:** $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v - v_0}{t - 0}$
- **instantaneous acceleration:** $a = \dfrac{\Delta v}{\Delta t} \quad (\Delta t \to 0)$



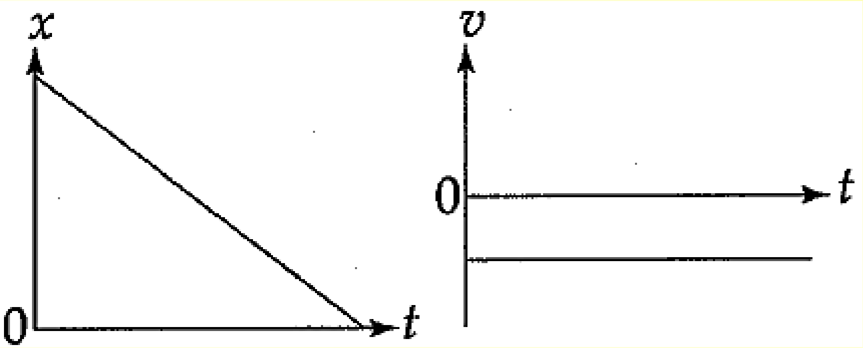
Kinematics with Graphs



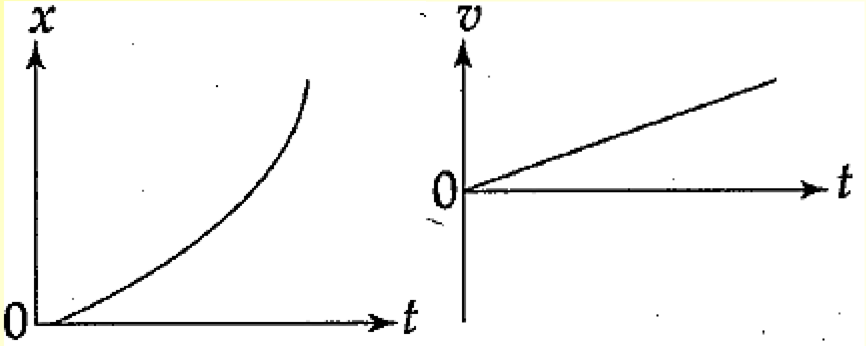
- $\triangle x = 0$
- $v = 0$
- $a = 0$



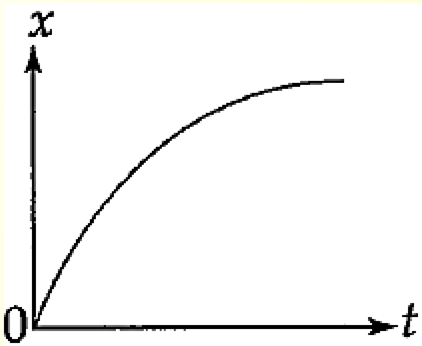
- $\triangle x > 0$, and $|\Delta x|$ ↑
- v = Const.
- $a = 0$

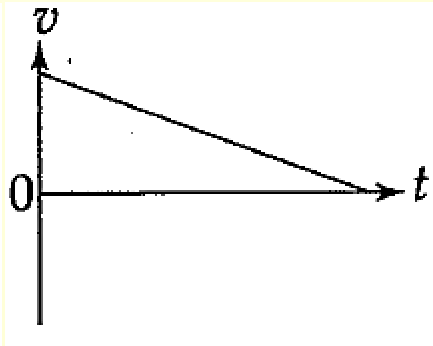


- $\triangle x < 0$, and $|\Delta x|$ ↑
- v = Const.
- $a = 0$

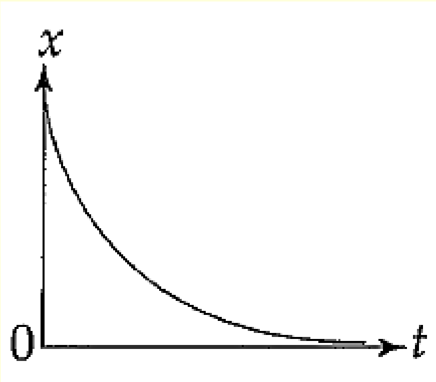


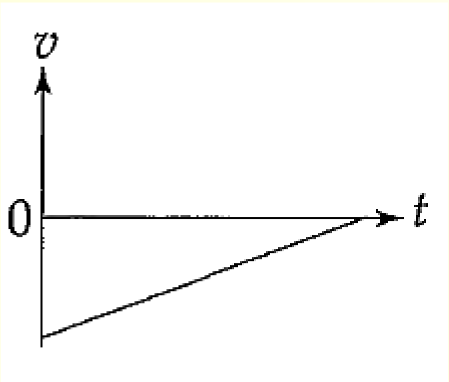
- $\triangle x > 0$, and $|\Delta x|$ ↑
- $v > 0$, and $|v|$ ↑
- $a > 0$ (accelerating)



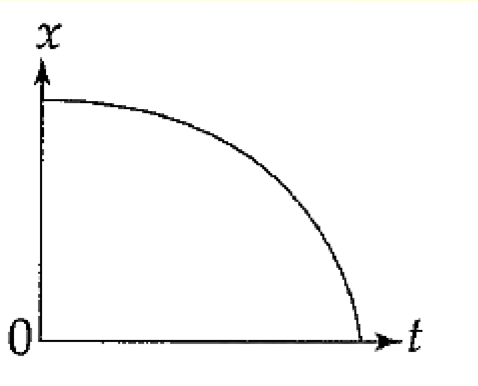


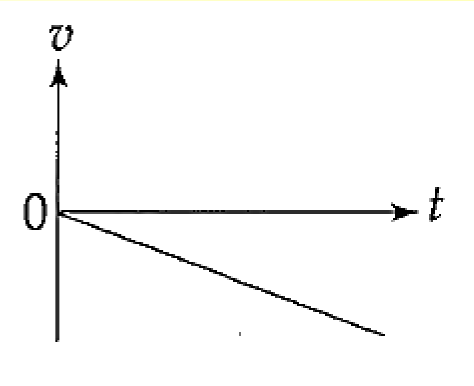
- **$\triangle x > 0$, and $|\Delta x|$ ↑**
- **$v > 0$, and $|v|$ ↓**
- **$a < 0$ (decelerating)**





- **$\triangle x < 0$, and $|\Delta x|$ ↑**
- **$v < 0$, and $|v|$ ↓**
- **$a > 0$ (decelerating)**

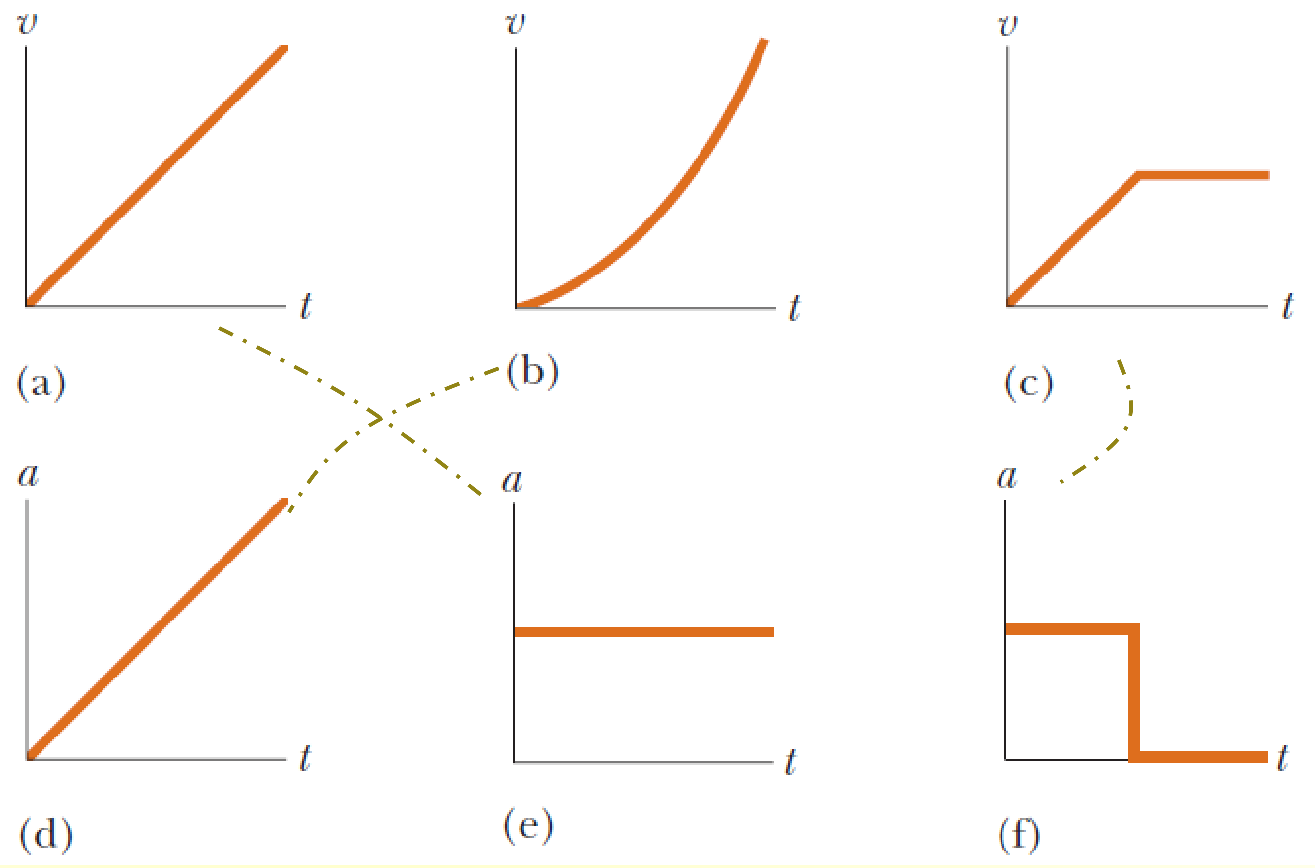




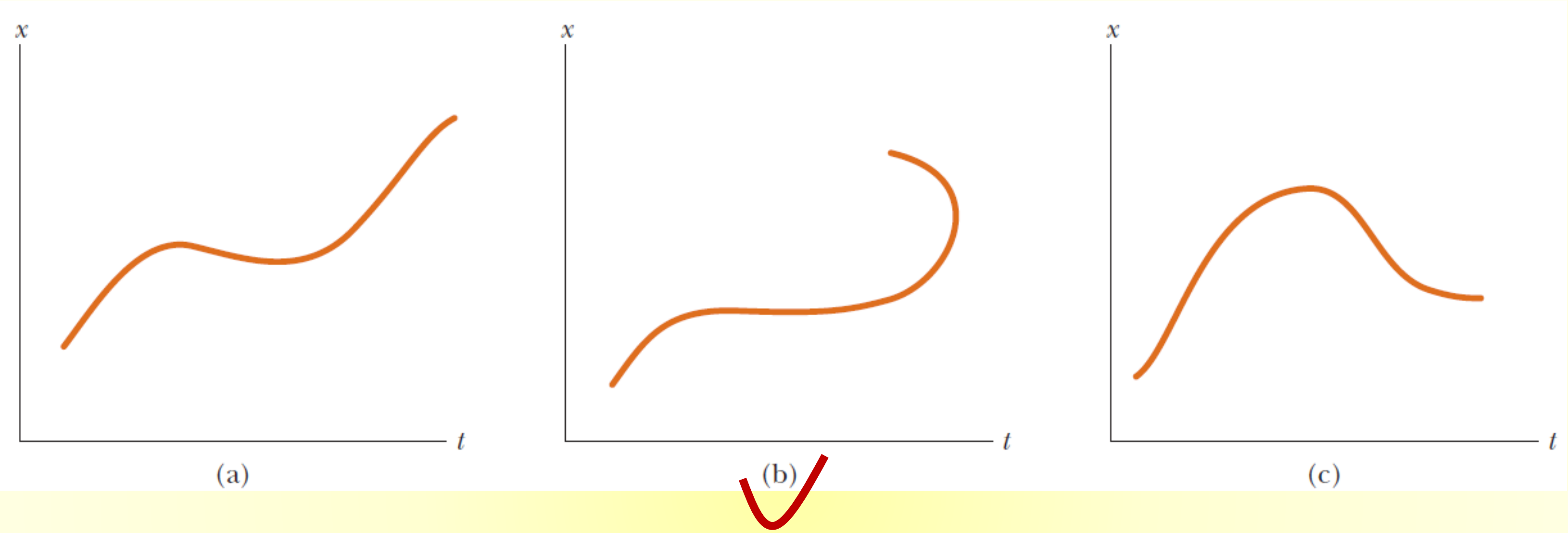
- **$\triangle x < 0$, and $|\Delta x|$ ↑**
- **$v < 0$, and $|v|$ ↑**
- **$a < 0$ (accelerating)**

Exercise:

Parts (a), (b) and (c) of Figure represent three graphs of the velocities of different objects moving in straight-line paths as functions of time. The possible accelerations of each object as functions of time are shown in parts (d), (e), and (f). Match each *v*-*t* graph with *a*-*t* graph that best describes the motion.



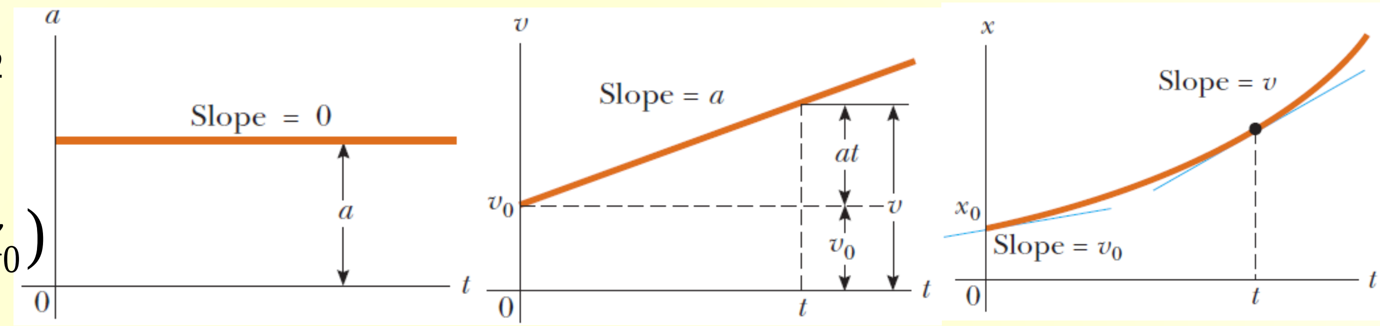
Exercise:



The three graphs in Figure above represent the position vs. time for objects moving along the *x*-axis. Which, if any, of these graphs is not physically possible?

Uniformly Accelerated Motion

- $v = v_0 + at$
- $x = x_0 + v_0 t + \frac{1}{2}at^2$
- $v^2 = v_0^2 + 2a(x - x_0)$
- $\bar{v} = \frac{v_0 + v}{2}$



Free Fall

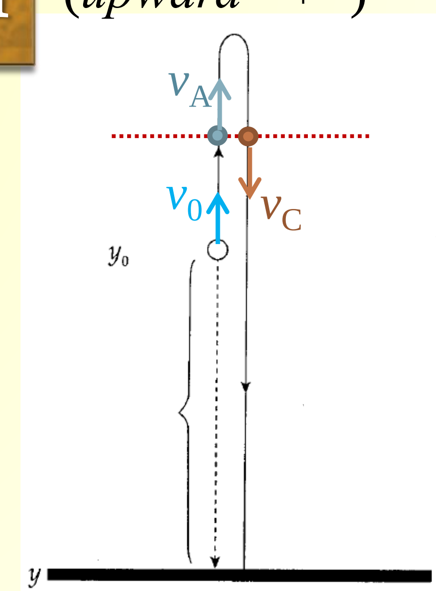
(*downward* "+")

- $v = gt$
- $y = y_0 + \frac{1}{2}gt^2$
- $\bar{v} = \frac{v}{2}$
- $v^2 = 2g(y - y_0)$

Upward Projection

(*upward* "+")

- $v = v_0 - gt$
- $y = y_0 + v_0 t - \frac{1}{2}gt^2$



Exercise:

1. A rock is dropped off a cliff and strikes the ground with an impact velocity of 30 m/s. How high was the cliff?

(A) 20 m (B) 30 m (C) 45 m (D) 60 m

2. A baseball is thrown straight upward. What is the ball's acceleration at its highest point?

(A) $\frac{1}{2}g$, downward

(B) g, downward

(C) $\frac{1}{2}g$, upward

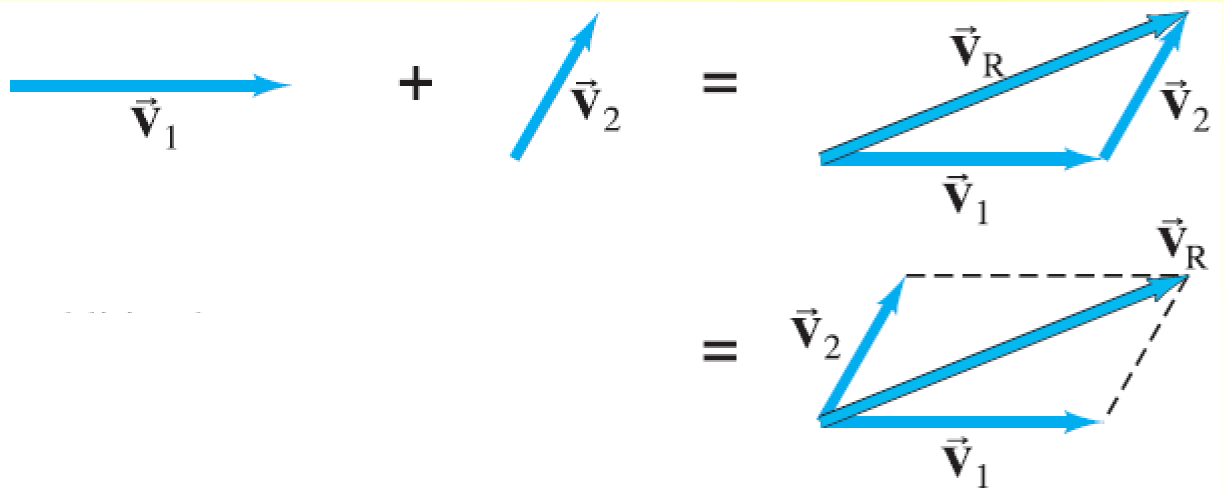
(D) g, upward

3. If an object is thrown straight upward with an initial speed of 8 m/s and takes 3 seconds to strike the ground, from what height was the object thrown? And what is the instantaneous velocity of the object when it strikes the ground?

$y_0 = 21$ m; $v = -22$ m/s

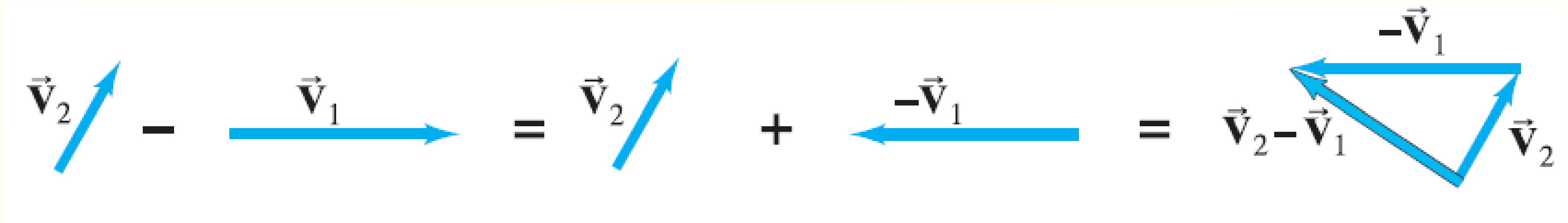
Two-Dimensional Motion

Addition of Vectors

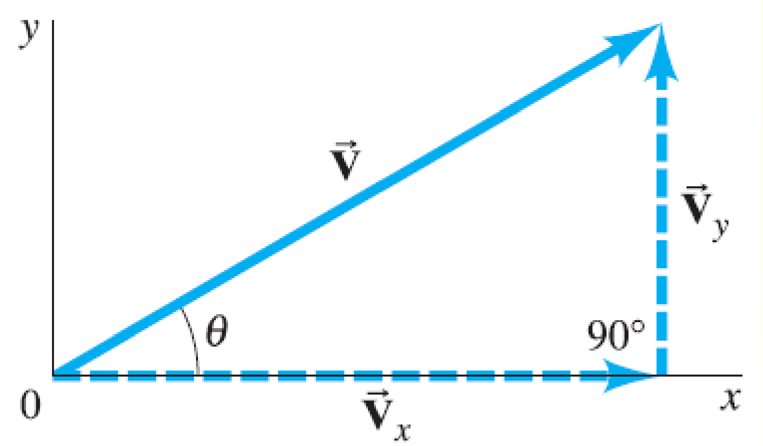


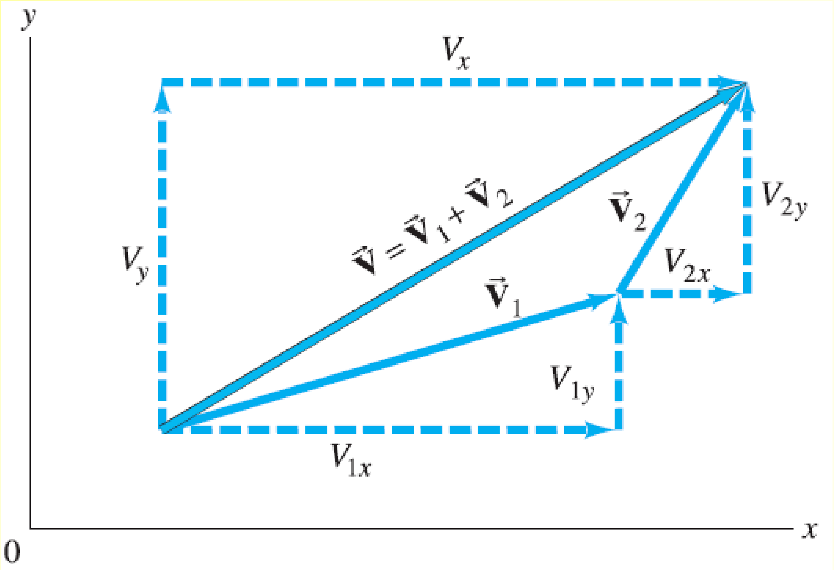
- **Tip-to-tail (triangle) method**
- **Parallelogram method**

Subtraction of Vectors



Components of a Vector



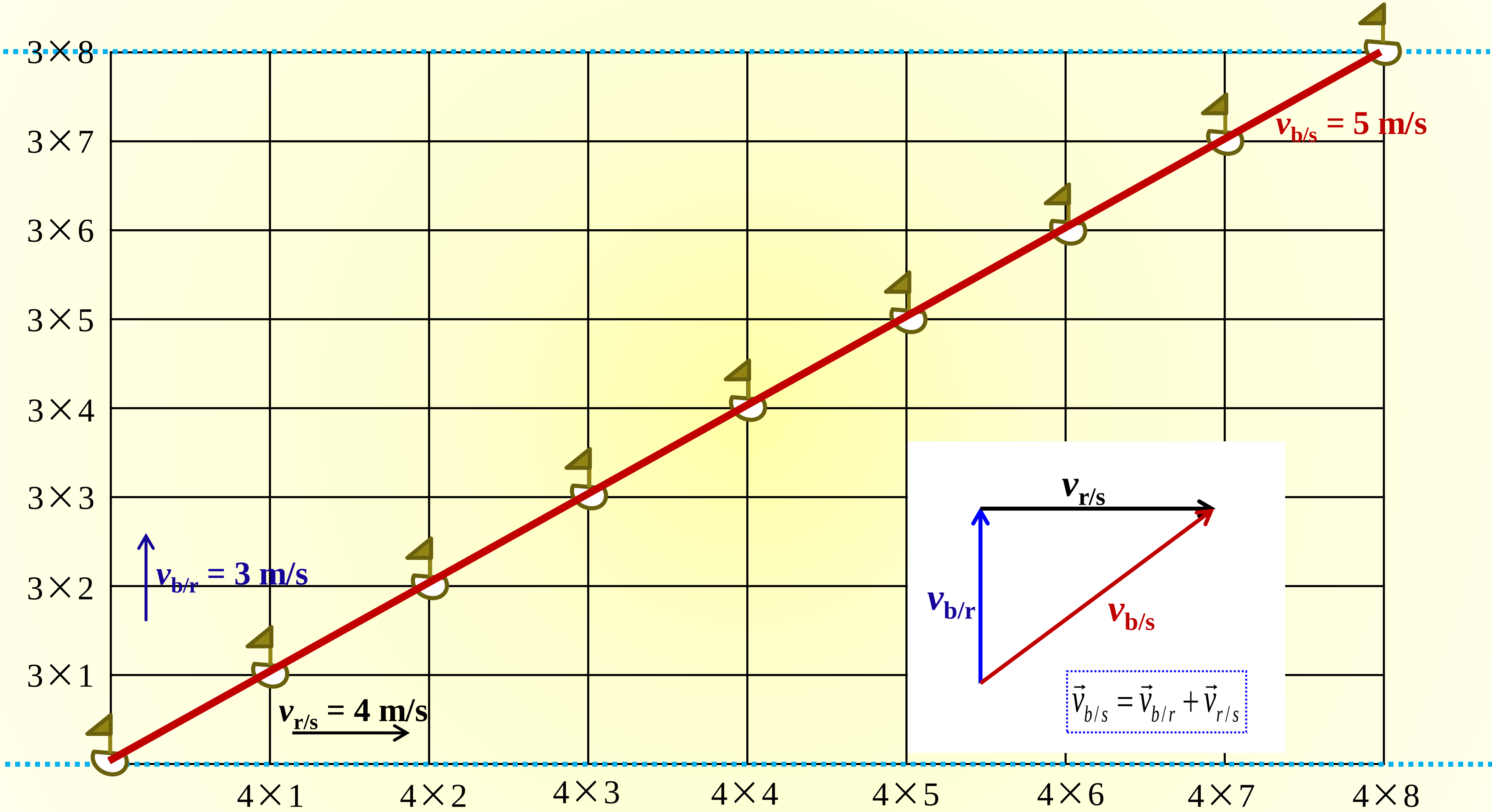


$$V_x = V\cos\theta \qquad V = \sqrt{V_x^2 + V_y^2}$$

$$V_y = V\sin\theta \qquad \tan\theta = \frac{V_y}{V_x}$$

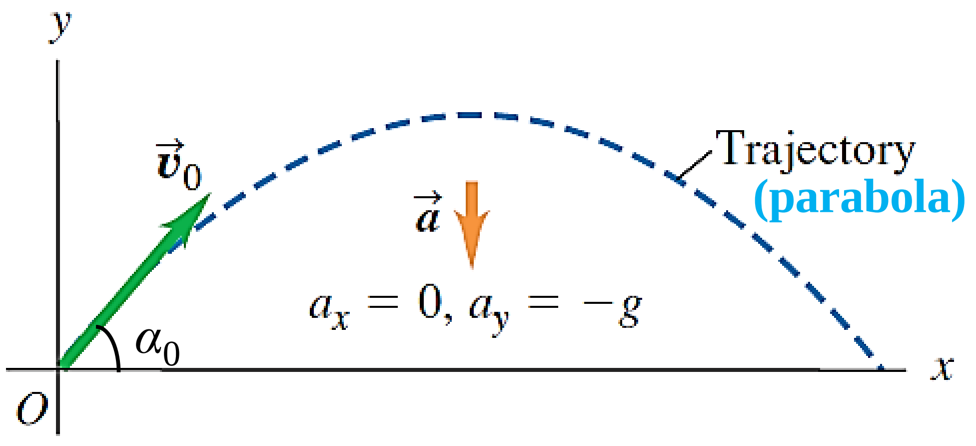
$$V_x = V_{1x} + V_{2x} \qquad V_y = V_{1y} + V_{2y}$$

Relative Motion



Projectile Motion

♠ **Component motions:**
independent;
simultaneous

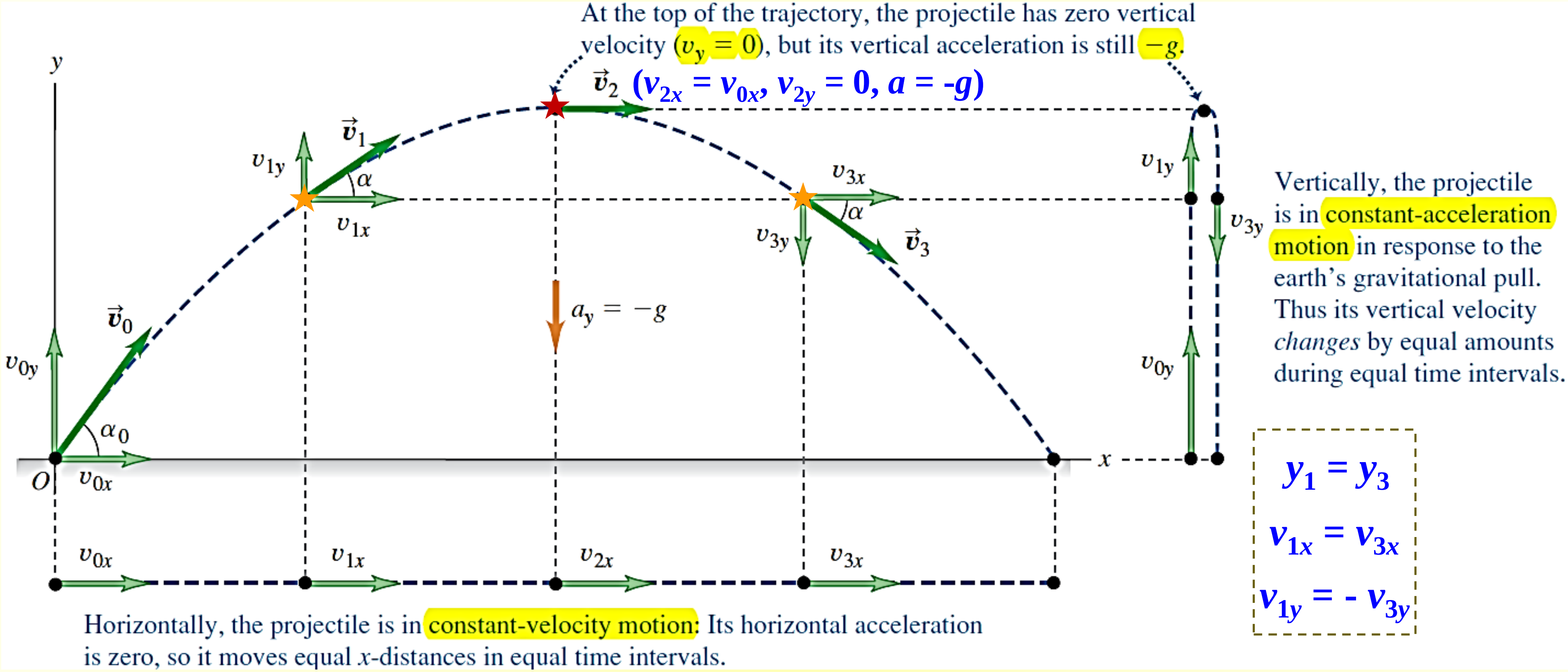


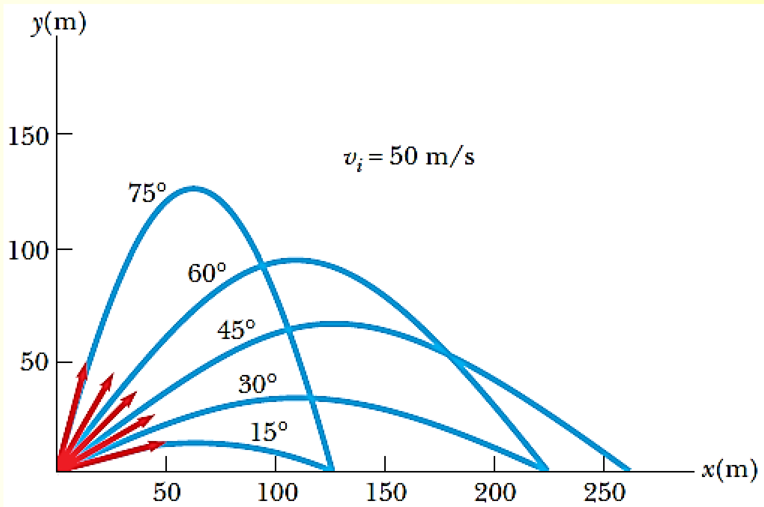
- ***x*-direction** (*right* "+"):

$$\left.\begin{array}{l} v_{0x} = v_0 \cos\alpha_0 \\ a_x = 0 \end{array}\right\} \Rightarrow$$ **uniform motion** $$\left\{\begin{array}{l} x = (v_0 \cos\alpha_0)t \quad [x_0 = 0] \\ v_x = v_0 \cos\alpha_0 \\ a_x = 0 \end{array}\right.$$

- ***y*-direction** (*upward* "+"):

$$\left.\begin{array}{l} v_{0y} = v_0 \sin\alpha_0 \\ a_y = -g \end{array}\right\} \Rightarrow$$ **uniformly accelerated motion due to gravity** $$\left\{\begin{array}{l} y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2 \quad [y_0 = 0] \\ v_y = v_0 \sin\alpha_0 - gt \\ a_y = -g \end{array}\right.$$





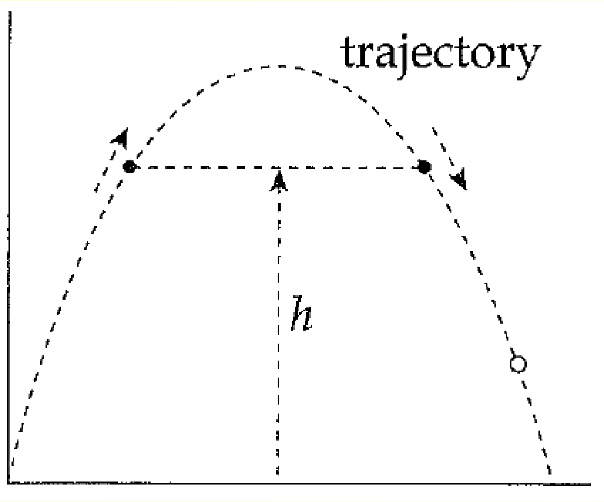
Note that complementary angles give the same horizontal range.

Exercise:

Consider a projectile moving in a parabolic trajectory under constant gravitational acceleration. Its initial velocity has magnitude v_0, and its launch angle (with the horizontal) is θ_0.

(a) Calculate the maximum height, *H*, of the projectile.

(b) Calculate the (horizontal) range, *R*, of the projectile.



(a) The maximum height of the projectile occurs at the time at which its vertical velocity drops to zero:

$$v_y = 0 \Rightarrow v_{0y} - gt = 0 \Rightarrow t = \frac{v_{0y}}{g}$$

The vertical displacement at this time can be solved as follows:

$$H = v_{0y}t - \frac{1}{2}gt^2 = \frac{v_{oy}^2}{2g} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

(b) The total flight time is: $t_{tot} = 2\dfrac{v_{0y}}{g}$

The horizontal displacement throughout the flight gives the projectile's range:

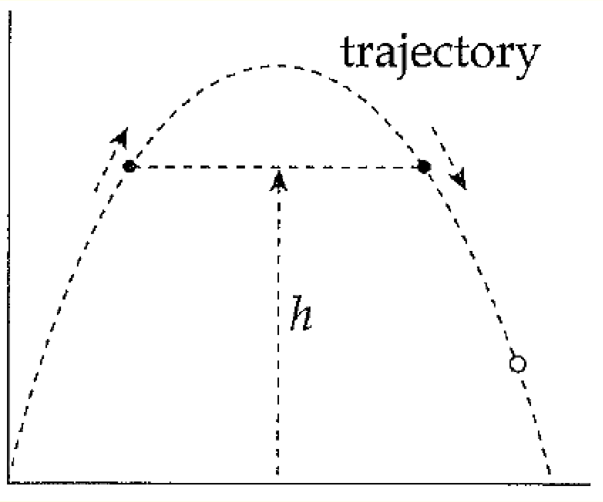
$$R = v_{0x}t_{tot} = \frac{2v_{0x}v_{0y}}{g} = \frac{v_0^2 \sin 2\theta_0}{g}$$

Exercise:

Consider a projectile moving in a parabolic trajectory under constant gravitational acceleration. Its initial velocity has magnitude v_0, and its launch angle (with the horizontal) is θ_0.

(c) For what value of θ_0 will the range be maximized?

(d) If $0 < h < H$, compute the time required for the projectile to pass through the two points shown in the figure.



(c) The range is: $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$

So when $\sin 2\theta_0$ is maximized, the range R will be maximized.

This occurs when $2\theta_0 = 90^o$, that is, when $\theta_0 = 45^o$.

(d) Set the vertical displacement equal to h and solve for the two values of t:

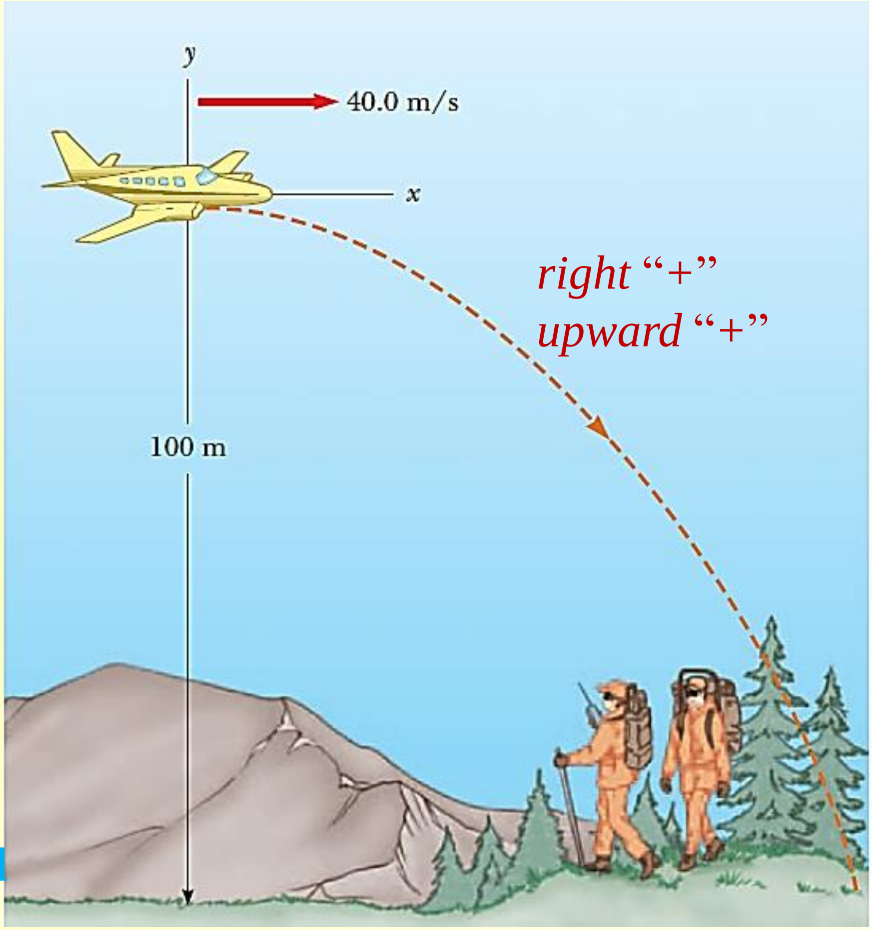
$$v_{0y}t - \frac{1}{2}gt^2 = h \;\Rightarrow\; \frac{1}{2}gt^2 - v_{0y}t + h = 0 \;\Rightarrow\; t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2gh}}{g}$$

So the time required to pass through the two points is:

$$\Delta t = \frac{v_{0y} + \sqrt{v_{0y}^2 - 2gh}}{g} - \frac{v_{0y} - \sqrt{v_{0y}^2 - 2gh}}{g} = \frac{2\sqrt{v_{0y}^2 - 2gh}}{g} = \frac{2\sqrt{(v_0 \sin\theta_0)^2 - 2gh}}{g}$$

Exercise:

An Alaskan rescue plane drops a package of emergency rations to a stranded hiker. The plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground. (a) Where does the package strike the ground relative to the point at which it was released? (b) What are the horizontal and vertical components of the velocity of the package just before it hits the ground?



(a) The time for the package to hit the ground can be computed as follows:

$$y = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow -\frac{1}{2}gt^2 = -100 \Rightarrow t = 2\sqrt{5}\ \text{s}$$

The range of the package when it hit the ground is:

$$x = v_{0x}t = (40 \times 2\sqrt{5})\ \text{m} = 80\sqrt{5}\ \text{m}$$

(b) The *x*-component of the velocity at the time of impact is: $v_x = v_{0x} = v_0 = 40.0\ \text{m/s}$

The *y*-component of the velocity at the time of impact is: $v_y = -gt = -(10 \times 2\sqrt{5})\ \text{m/s} = -20\sqrt{5}\ \text{m/s}$

Motion in a Circle

♣ Description of circular motion:

- **linear velocity** (vector)

$$v = \frac{\Delta s}{\Delta t}$$

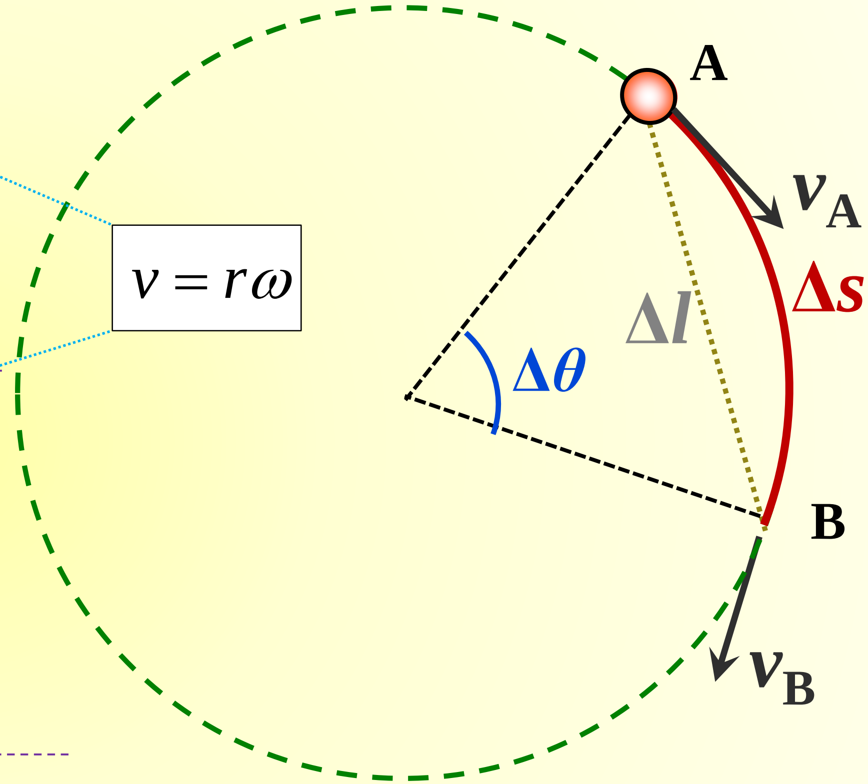
SI unit: m/s

$$v = r\omega$$

- **angular velocity** (vector)

$$\omega = \frac{\Delta \theta}{\Delta t}$$

SI unit: rad/s

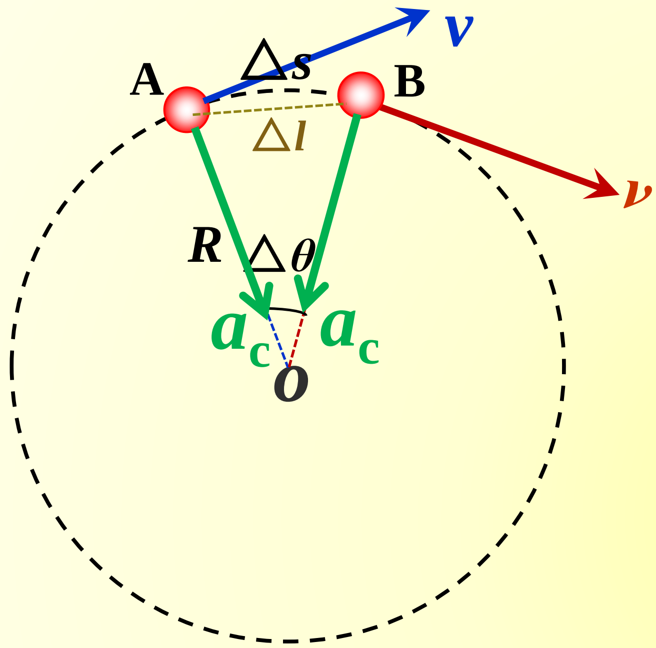


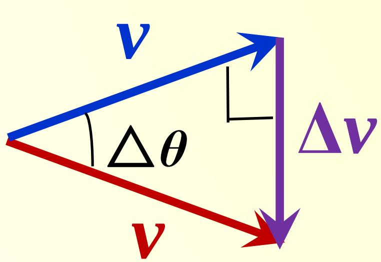
- **period** T **SI unit: s**

- **frequency** f **SI unit: Hz**

$$f = \frac{1}{T}$$

♣ Uniform Circular Motion (*constant speed* *along a circular path*)





$$\Delta t \to 0,\ \Delta\theta \to 0$$

$$\Delta v \perp v$$

$$a \perp v$$

The magnitude of centripetal acceleration

$$\frac{\Delta v}{v} = \frac{\Delta l}{R}$$

$$\Delta\theta \to 0,\ \Delta l = \Delta s$$

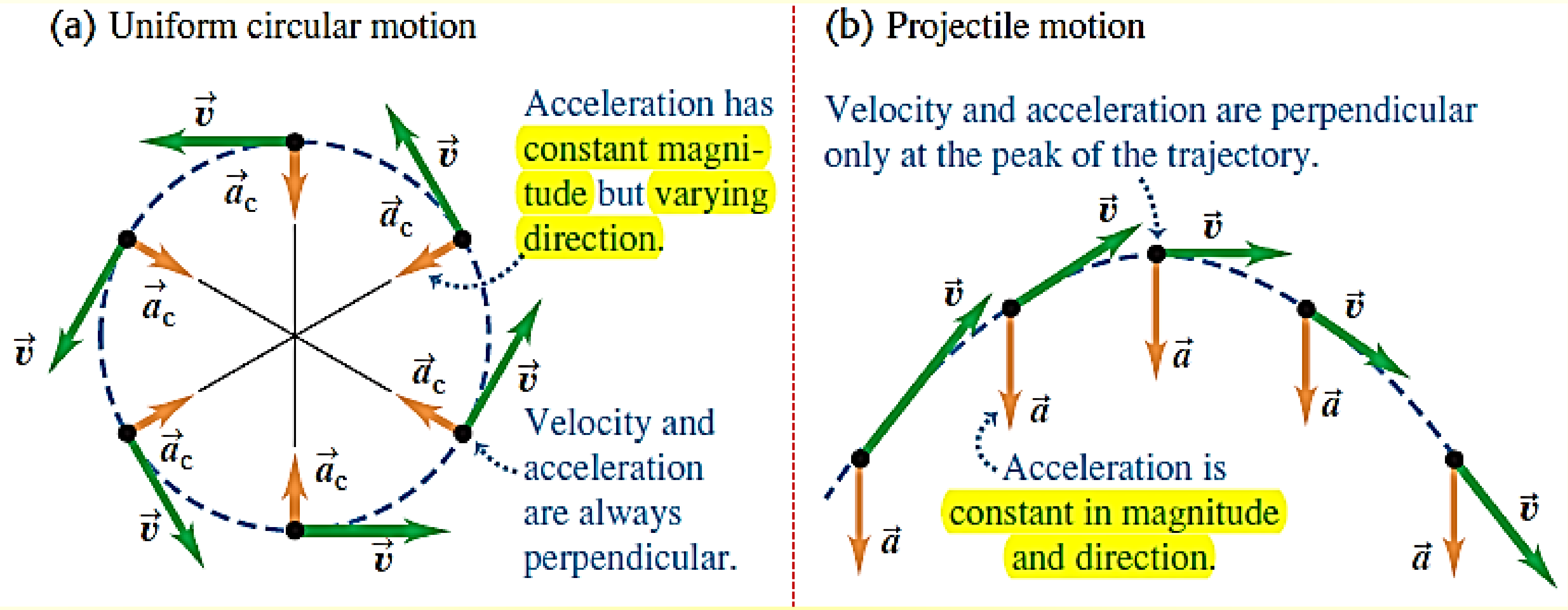
$$\Delta v = \frac{v}{R}\cdot \Delta s$$

$$a_c = \frac{\Delta v}{\Delta t}\ (\Delta t \to 0)$$

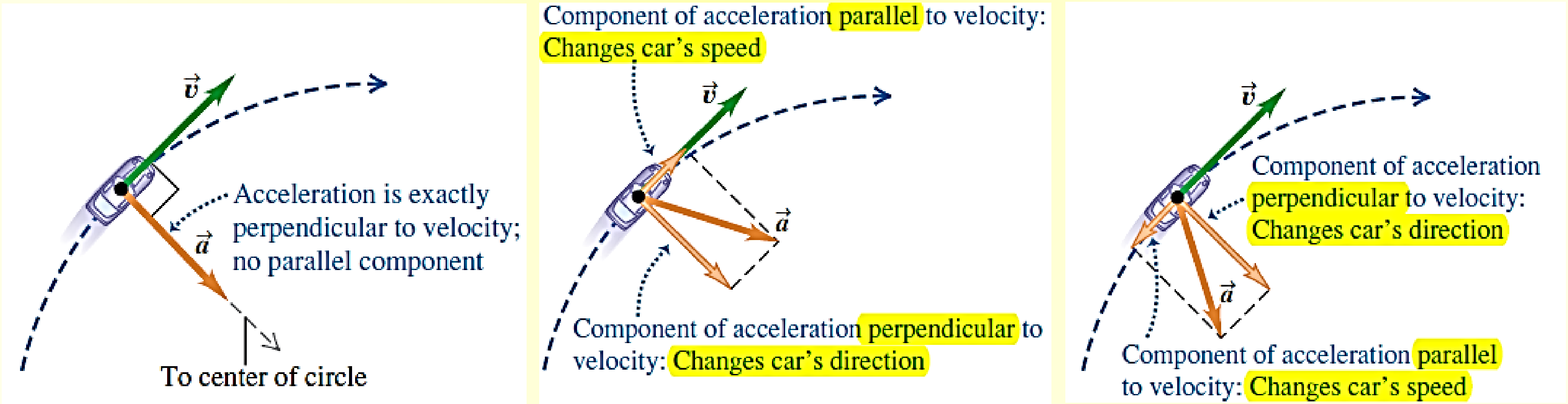
$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{4\pi^2 R}{T^2}\ (T = \frac{2\pi R}{v})$$

$$= R\omega^2\ (\omega = \frac{2\pi}{T} = \frac{v}{R})$$



♣Nonuniform Circular Motion (*speed varying along a circular path*)



Uniform circular motion

$$\vec{a} = \vec{a}_c$$

Speeding up along a circular path

$$\vec{a} = \vec{a}_c + \vec{a}_{\tan}$$

Slowing down along a circular path

$$\vec{a} = \vec{a}_c + \vec{a}_{\tan}$$

Exercise:

1. An object moves at constant speed in a circular path. Which of the following statements is true? Select two answers.

(A) The velocity is changing.

(B) The velocity is constant.

(C) The magnitude of acceleration is constant.

(D) The magnitude of acceleration is changing.

2. Suppose that the particle experiences four times the acceleration at the bottom of the loop as it does at the top of the loop. Compared to its speed at the top of the loop, is its speed at the bottom of the loop (i) $\sqrt{2}$ times as great; (ii) 2 times as great; (iii) $2\sqrt{2}$ times as great; (iv) 4 times as great; or (v) 16 times as great?