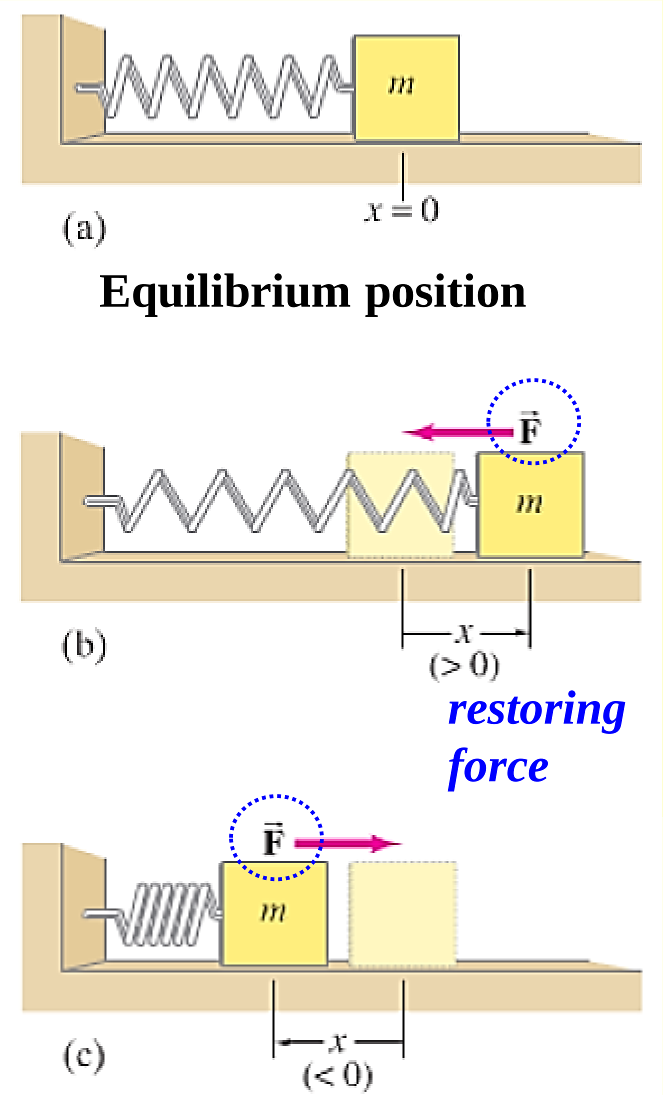
Chap7 Oscillations

Simple Harmonic Motion (SHM)



- A **restoring force** always pushes or pulls the object toward the equilibrium position.

Hooke's Law

$$F = -kx$$

- **Simple harmonic motion** occurs when the net force along the direction of motion obeys Hooke's law---when the net force is proportional to the displacement from the equilibrium point and is always directed toward the equilibrium point.

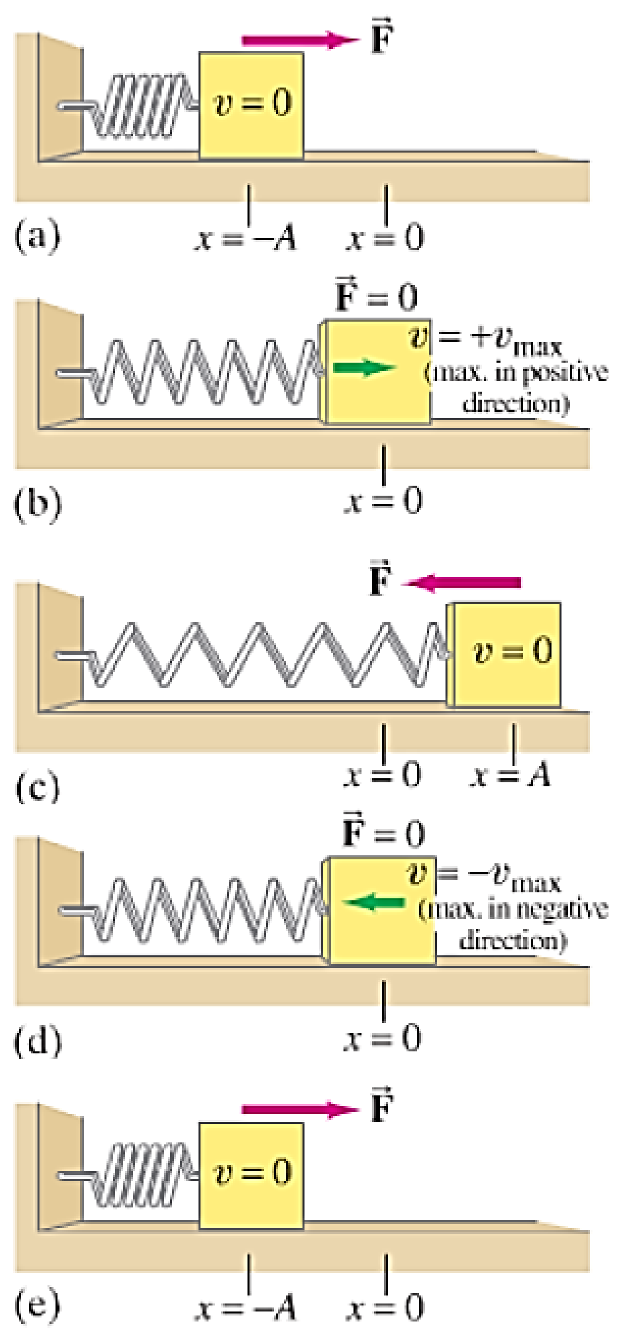
➢ Describing simple harmonic motion:

- **amplitude A:** the maximum distance of the object from its equilibrium position.
- **period T:** the time to move through one complete cycle of motion.
- **frequency f:** the number of complete cycles or vibrations per second.

$$f = \frac{1}{T}$$

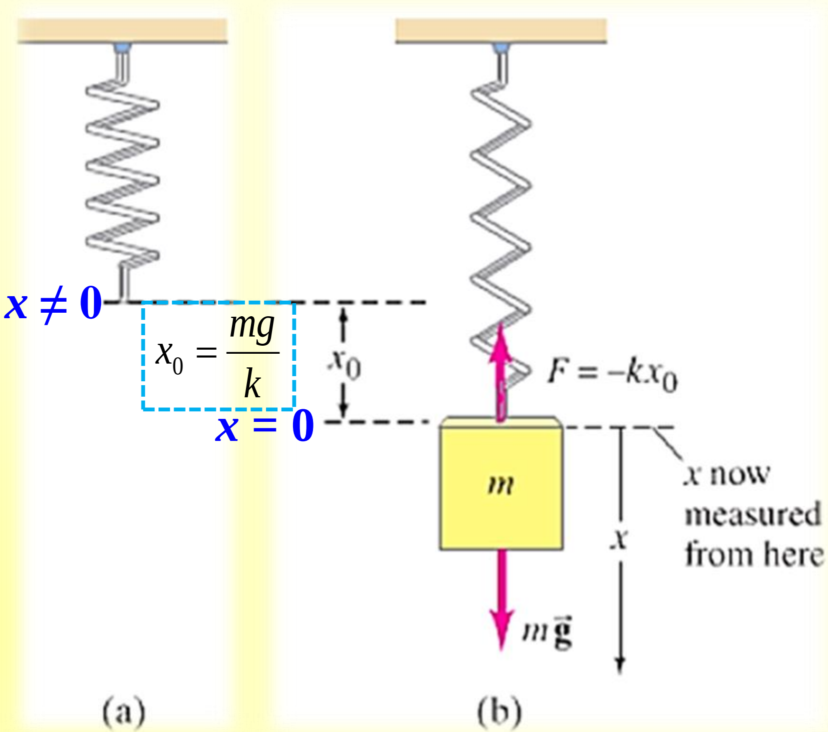
- **angular frequency ω:** the number of radians per second.

$$\omega = 2\pi f$$



Simple harmonic motion (SHM)

Simple harmonic oscillator (SHO)



【Exercise】

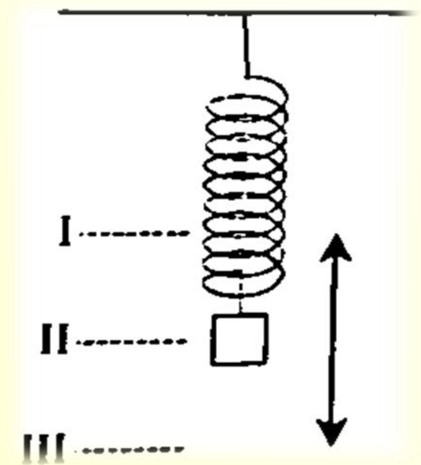
Which of the following represent a simple harmonic oscillator:

(a) $F = -0.5x^2$, **(b) $F = -2.3y$,**

(c) $F = 8.6x$, **(d) $F = -4\theta$**

【Exercise】

The diagram on the right shows a spring-mass system in oscillation and identifies three key points in the motion. Position I and III are the maximum displacements, and position II is the equilibrium position.



Q1. At which position(s) is the kinetic energy greatest?

(A) I only
(B) II only
(C) III only
(D) I and III only

Q2. At which position(s) is the acceleration the greatest?

(A) I only
(B) II only
(C) III only
(D) I and III only

Energy in the SHO

- ✓ **Spring potential energy:**

$$U_s = \frac{1}{2}kx^2$$

- ✓ **Total energy of SHO:**

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

at the extreme points,

x = -A (or A), v = 0

$$E = \frac{1}{2}m(0)^2 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2$$

$$E \propto A^2$$

at the equilibrium point,

x = 0, $v = v_{max}$

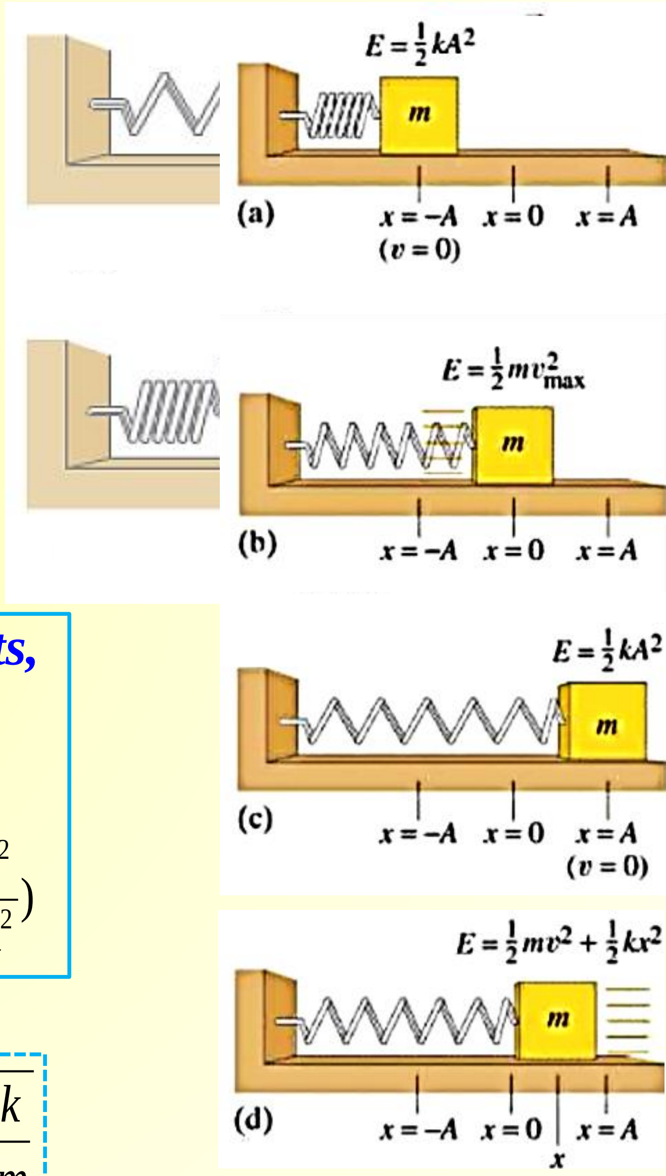
$$E = \frac{1}{2}mv_{max}^2 + \frac{1}{2}k(0)^2 = \frac{1}{2}mv_{max}^2$$

at the intermediate points,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad = \frac{1}{2}kA^2$$

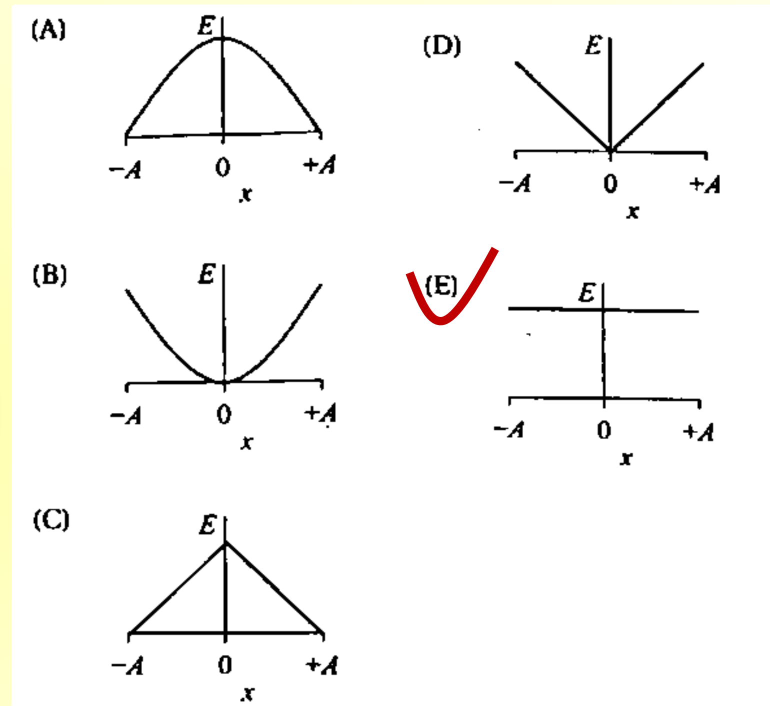
$$v^2 = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right) = v_{max}^2\left(1 - \frac{x^2}{A^2}\right)$$

$$v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}},\quad v_{max} = A\sqrt{\frac{k}{m}}$$

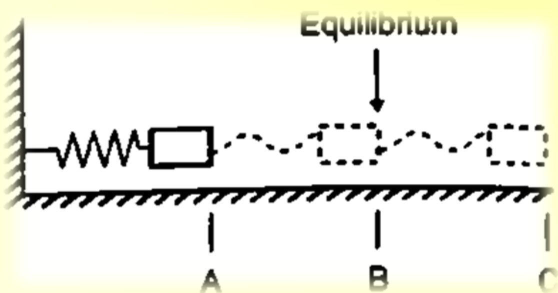


【Exercise】

Q1. Which graph correctly depicts the total energy during an oscillation?



Q2. A mass vibrates on an ideal spring as shown. The total energy of the spring is 100 J. What is the kinetic energy of the mass at point B?



(A) 25 J

(B) 50 J

(C) 75 J

(D) 100 J

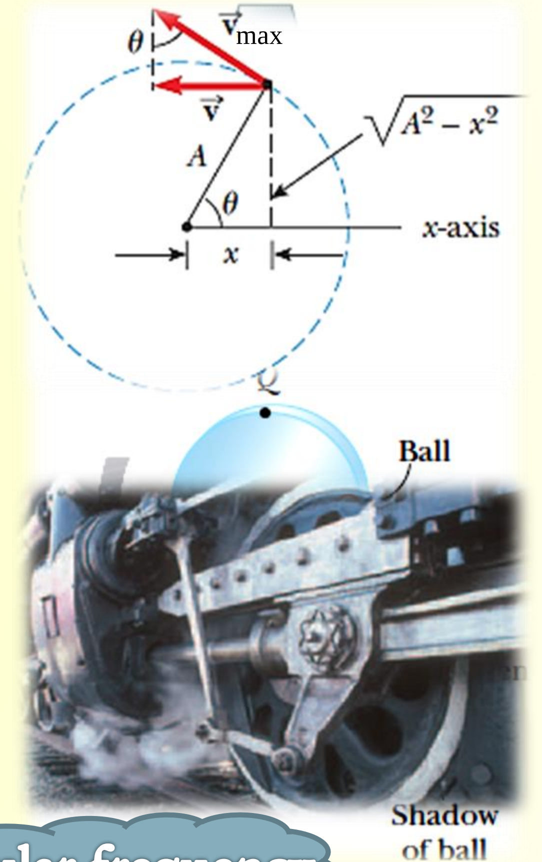
Comparing SHM with Uniform Circular Motion

- ✓ **Simple harmonic motion:** $$v = v_{max}\sqrt{1-\frac{x^2}{A^2}}$$

- ✓ **Shadow of the ball on uniform circular motion:**

$$\left.\begin{aligned} \sin\theta &= \frac{v}{v_{max}} \\ \sin\theta &= \frac{\sqrt{A^2-x^2}}{A} \end{aligned}\right\} \Rightarrow v = v_{max}\sqrt{1-\frac{x^2}{A^2}}$$

- *Thus, the shadow moves with simple harmonic motion.*



The Period, Frequency & Angular Frequency of SHM

$$\left.\begin{aligned} v_{max} = \frac{2\pi A}{T} &\Rightarrow T = \frac{2\pi A}{v_{max}} \\ \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2 &\Rightarrow v_{max} = A\sqrt{\frac{k}{m}} \end{aligned}\right\} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

angular frequency

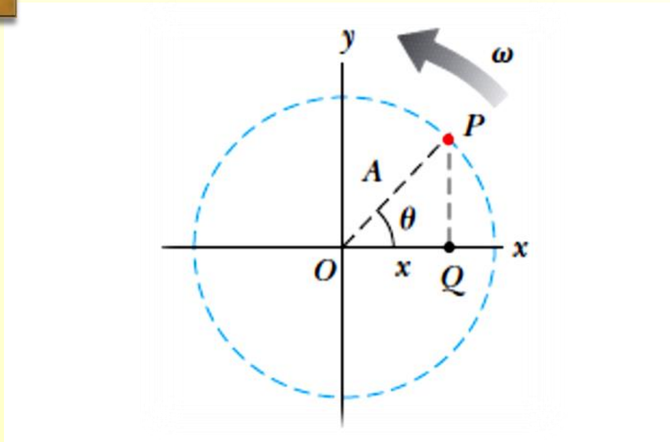
Relationship of *x-t*, *v-t* & *a-t* on SHM

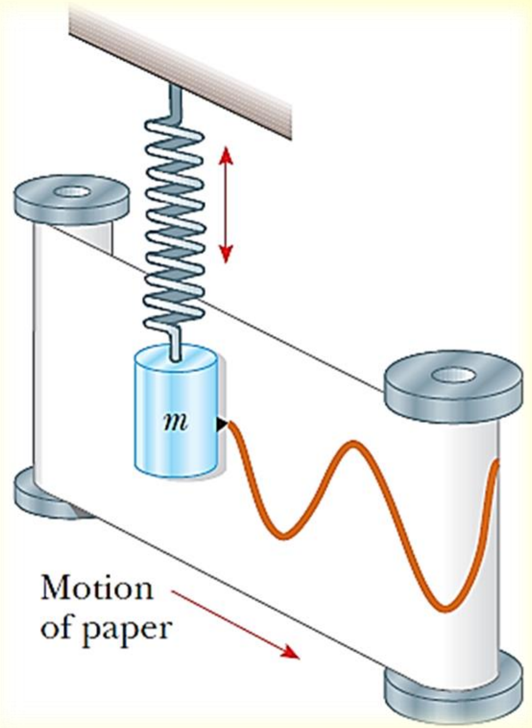
- The circle made by the ball is referred to as the *reference circle* for the projected SHM.

$$\left.\begin{array}{l} x = A\cos\theta \\ \theta = \omega t \end{array}\right\} \Rightarrow \boxed{x = A\cos(\omega t)}$$

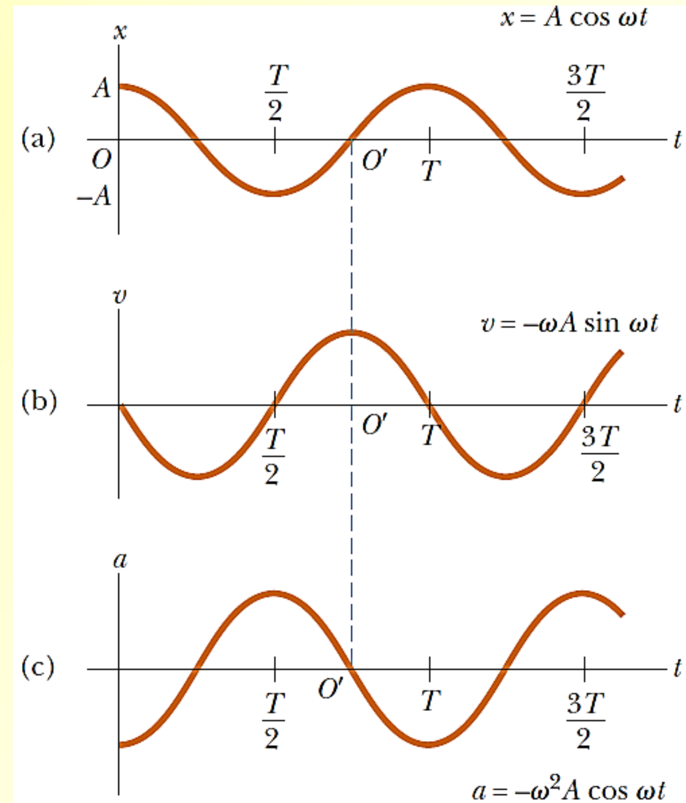
$$\boxed{v = -A\omega\sin(\omega t)}$$

$$\boxed{a = -A\omega^2\cos(\omega t)}$$





- *An experimental apparatus for demonstrating SHM. A pen attached to the oscillating objects traces out a sinusoidal wave on the moving chart paper.*



【Quick Quiz】

1. An object of mass m is attached to a horizontal spring, stretched to a displacement A from equilibrium and released, undergoing harmonic oscillations on a frictionless surface with period T_0. The experiment is then repeated with a mass of 4m. What's the new period of oscillation?

   (A) $2T_0$ (B) T_0 (C) $T_0/2$ (D) $T_0/4$

2. Consider the situation above. The subsequent total mechanical energy of the object with mass 4m is

   (A) greater than (B) less than (C) equal to

   the original total mechanical energy.

3. If the amplitude of a system moving in simple harmonic motion is doubled, which of the following quantities doesn't change?

   (A) total energy (B) maximum speed

   (C) maximum acceleration (D) period

The Simple Pendulum

$$F = -mg\sin\theta$$

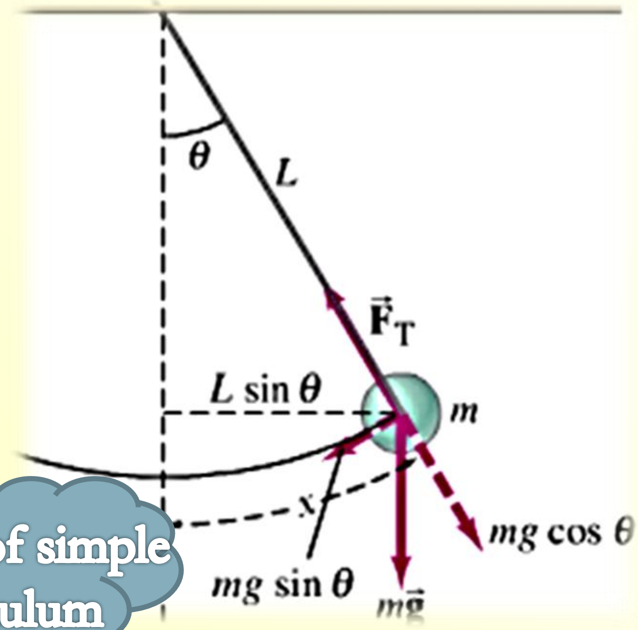
When $\theta \le 10°, \sin\theta \approx \theta$

$$\Rightarrow F \approx -mg\theta$$

$$\theta = \frac{x}{L}$$

$$F \approx -\frac{mg}{L}x \qquad \frac{mg}{L} = k$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$



Period of simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

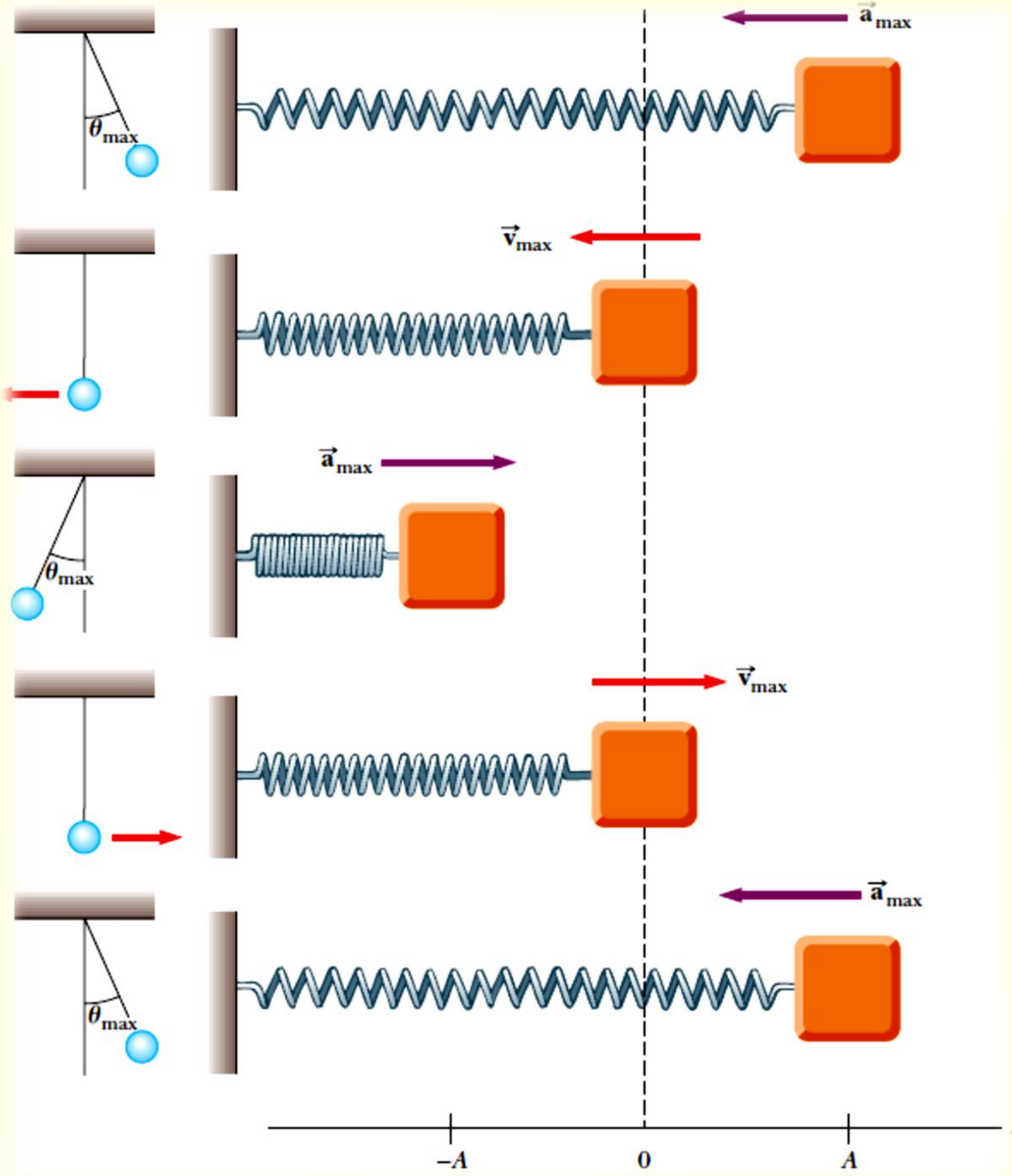
Frequency of simple pendulum

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Angular frequency of simple pendulum

$$\omega = 2\pi f = \sqrt{\frac{g}{L}}$$

pendulum clock



- *Simple harmonic motion* for an object-spring system, and its analogy, the motion of a *simple pendulum*.

【Quick Quiz】

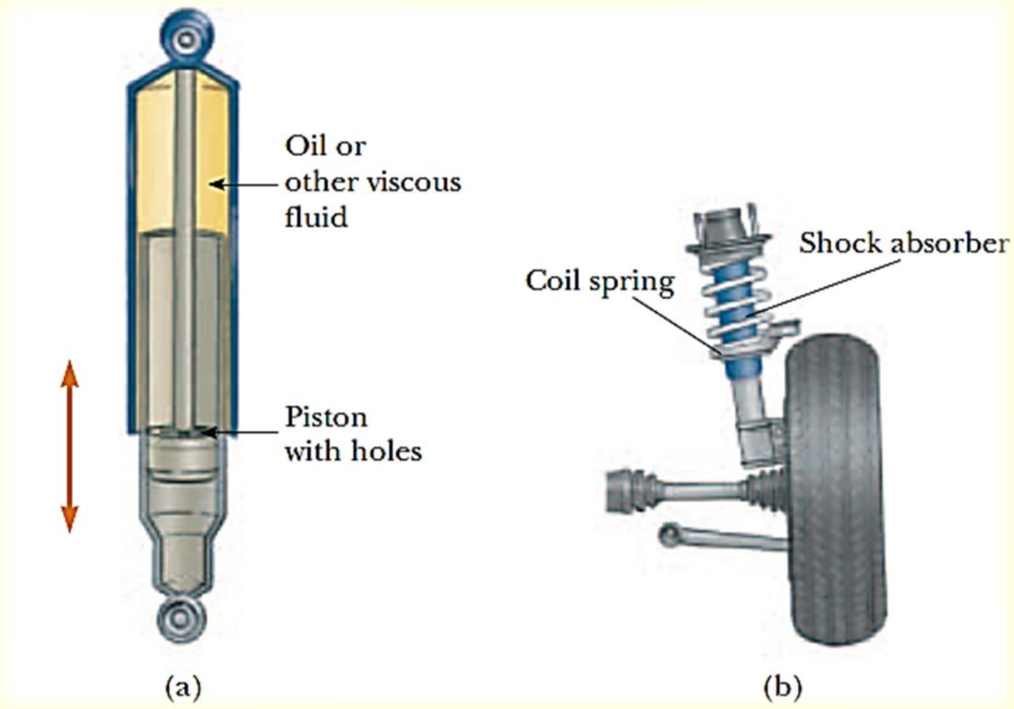
1. A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is measured. If the elevator moves with constant velocity, does the period (a) increase, (b) decrease, or (c) remain the same? If the elevator accelerates upward, does the period (a) increase, (b) decrease, or (c) remain the same?

2. A pendulum clock depends on the period of a pendulum to keep correct time. Suppose a pendulum clock is keeping correct time and then Dennis the Menace slides the bob of the pendulum downward on the oscillating rod. Does the clock run (a) slow, (b) fast, or (c) correctly?

3. The period of a simple pendulum is measured to be T on Earth. If the same pendulum was set in motion on the Moon, would its period be (a) less than T, (b) greater than T, or (c) equal to T?

Damped Oscillations

- The oscillation with the *decreasing amplitude* is called **damped oscillation**.
- Damping plays a beneficial role in the oscillations of an automobile's suspension system.



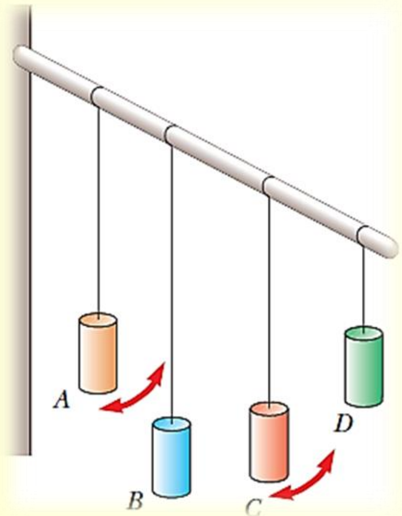
(a) *A shock absorber. The oil is squeezed through holes between the piston and the chamber, causing a damping of the piston's oscillations.*

(b) *One type of automotive suspension system*

Forced Oscillations & Resonance

- **Natural frequency:**

When the object is simply displaced from equilibrium and then left alone, the object will oscillate with a natural frequency f_0 (determined by m, k , b).



- **Forced Oscillation:**

the damped oscillating object having some natural frequency f_0 is pushed back and forth by ***a periodic force*** with frequency f, and the object vibrates at the frequency f. This type of motion is referred to as a forced oscillation or a driven oscillation.

- **Resonance:**

When the frequency f of forced oscillations on an object matches the object's natural frequency f_0, a ***dramatic increase in amplitude*** occurs. This phenomenon is called resonance.

Applying Physics Bungee Jumping

A bungee cord can be modeled as a spring. If you go bungee jumping, you will bounce up and down at the end of the elastic cord after your dive off a bridge (Fig. 1). Suppose you perform a dive and measure the frequency of your bouncing. You then move to another bridge, but find that the bungee cord is too long for dives off this bridge. What possible solutions might be applied? In terms of the original frequency, what is the frequency of vibration associated with the solution?


Telegraph Colour Library/FPG International/Getty Images

Figure1 Bungee jumping from a bridge.

Explanation There are two possible solutions: Make the bungee cord smaller or fold it in half. The latter would be the safer of the two choices, as we'll see. The force exerted by the bungee cord, modeled as a spring, is proportional to the separation of the coils as the spring is extended. First, we extend the spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Therefore, it takes twice as much force to stretch the half-spring through the same displacement, so the half-spring has a spring constant twice that of the complete spring. The folded bungee cord can then be modeled as two half-springs in parallel. Each half has a spring constant that is twice the original spring constant of the bungee cord. In addition, an object hanging on the folded bungee cord will experience two forces—one from each half-spring. As a result, the required force for a given extension will be four times as much as for the original bungee cord. The effective spring constant of the folded bungee cord is therefore four times as large as the original spring constant. Because the frequency of oscillation is proportional to the square root of the spring constant, your bouncing frequency on the folded cord will be twice what it was on the original cord.

This discussion neglects the fact that the coils of a spring have an initial separation. It's also important to remember that a shorter coil may lose elasticity more readily, possibly even going beyond the elastic limit for the material, with disastrous results. Bungee jumping is dangerous—discretion is advised!