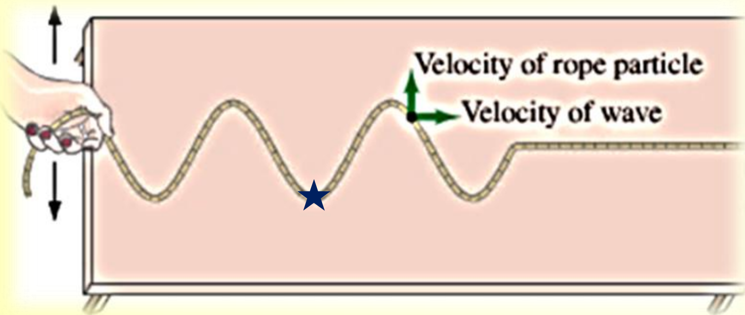
Chap8 Waves

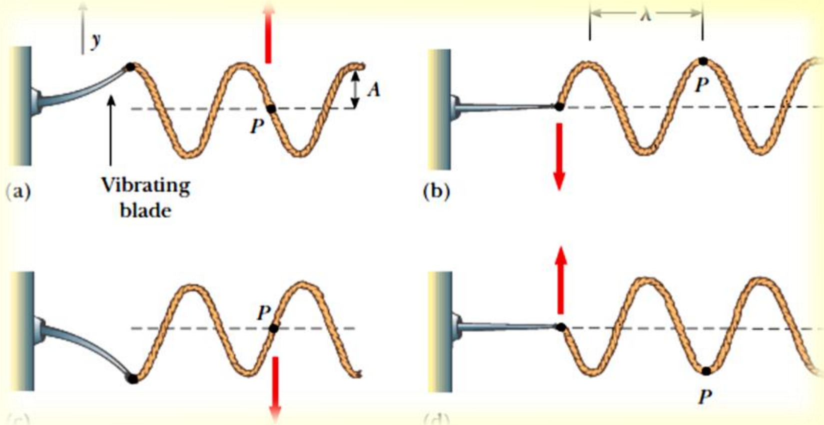
Waves

waves: ***the motion of disturbance,***
not carrying matter along

- **Mechanical Waves:**
  - some source of disturbance
  - a medium that can be disturbed
  - adjacent portions of the medium can influence each other

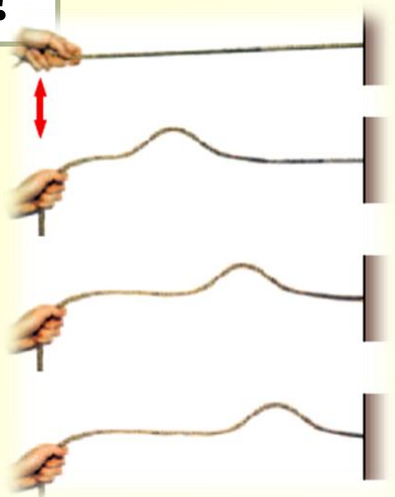


Traveling waves can transport energy & momentum!

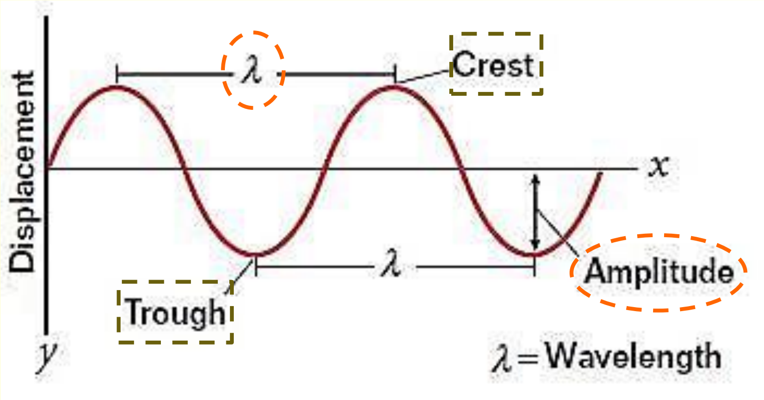


- a continuous /periodic wave

- a wave pulse



➢ Describing waves:



amplitude A: *the height of a crest/trough*

wavelength λ: *the distance between two successive crests/troughs*

period T: *the time elapsed between two successive crests/troughs passing the same point*

frequency f: *the number of crests/troughs that pass a given point per second*

wave velocity v: $\boxed{v = \frac{\lambda}{T}}$ $\boxed{v = \lambda f}$

- v --- determined by the medium
- f --- determined by the wave source
- Wave speed on a stretched string: $\boxed{v = \sqrt{\frac{F_T}{\mu}}}$
  - ✓ μ --- *the linear mass density* (kg/m)
  - ✓ F_T --- *the tension in the stretched string*

【Exercise】

1. Waves are created on the surface of the water in a pool by a device that dips up and down 20.0 times per minute. The velocity of the resultant wave is 3.00 m/s.
   a. State the frequency, period and wavelength of the observed wave.
   b. What happens to the value of the velocity and wavelength if the frequency of the source is doubled?
   a. f = 1/3 Hz, T = 3.00 s, λ = 9.00 m
   b. v = 3.00 m/s, λ = 4.50 m
2. A rope of length 5 m is stretched to a tension of 80 N. If its mass is 1 kg, at what speed would a 10 Hz transverse wave travel down the string?
   (A) 2 m/s (B) 5 m/s (C) 20 m/s
   (D) 50 m/s (E) 200 m/s

Types of waves

transverse waves

(propagate in *solid*) **S waves**

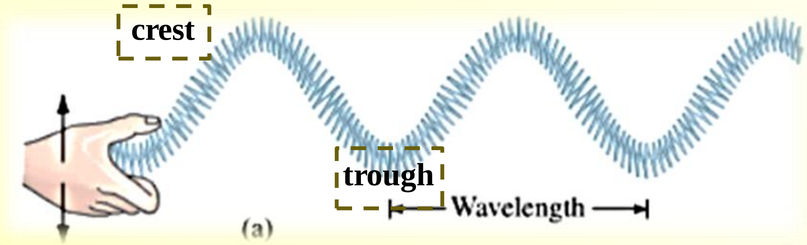
longitudinal waves

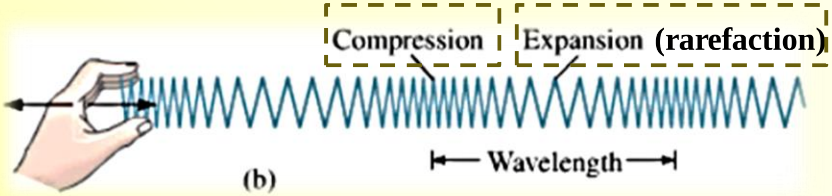
(propagate in *solid, liquid & gas*) **P waves**

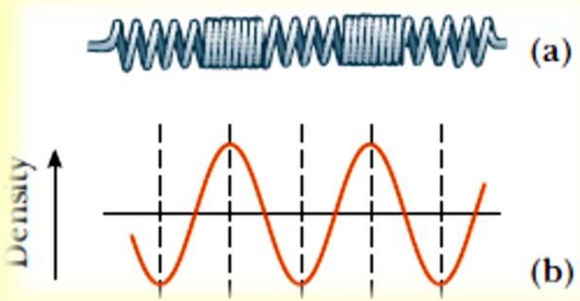
surface waves **L waves**

seismic waves

$v_P > v_S > v_L$

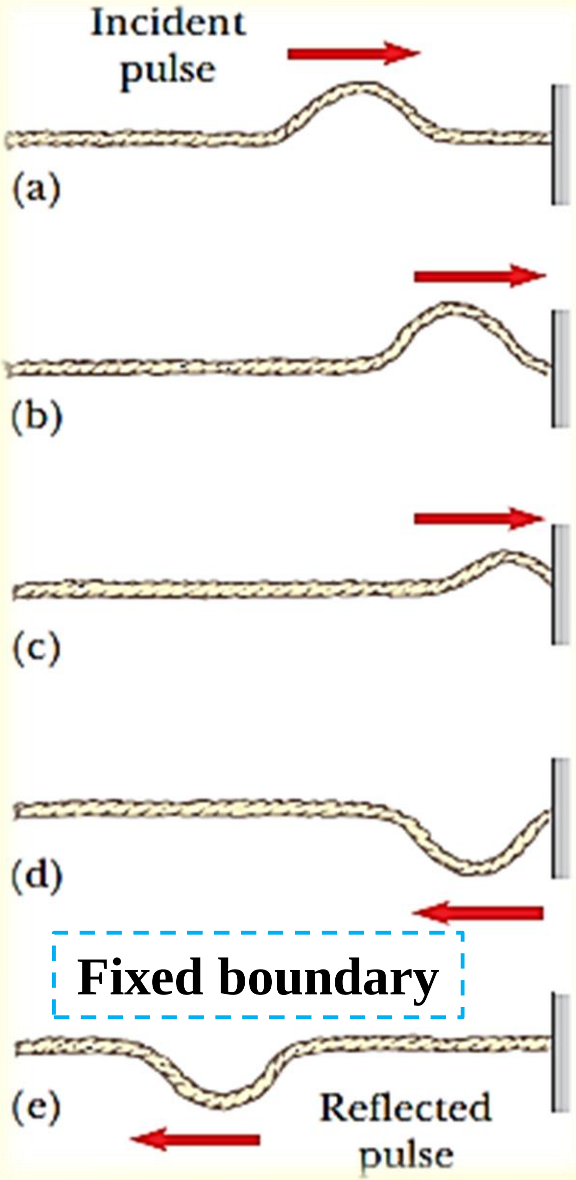


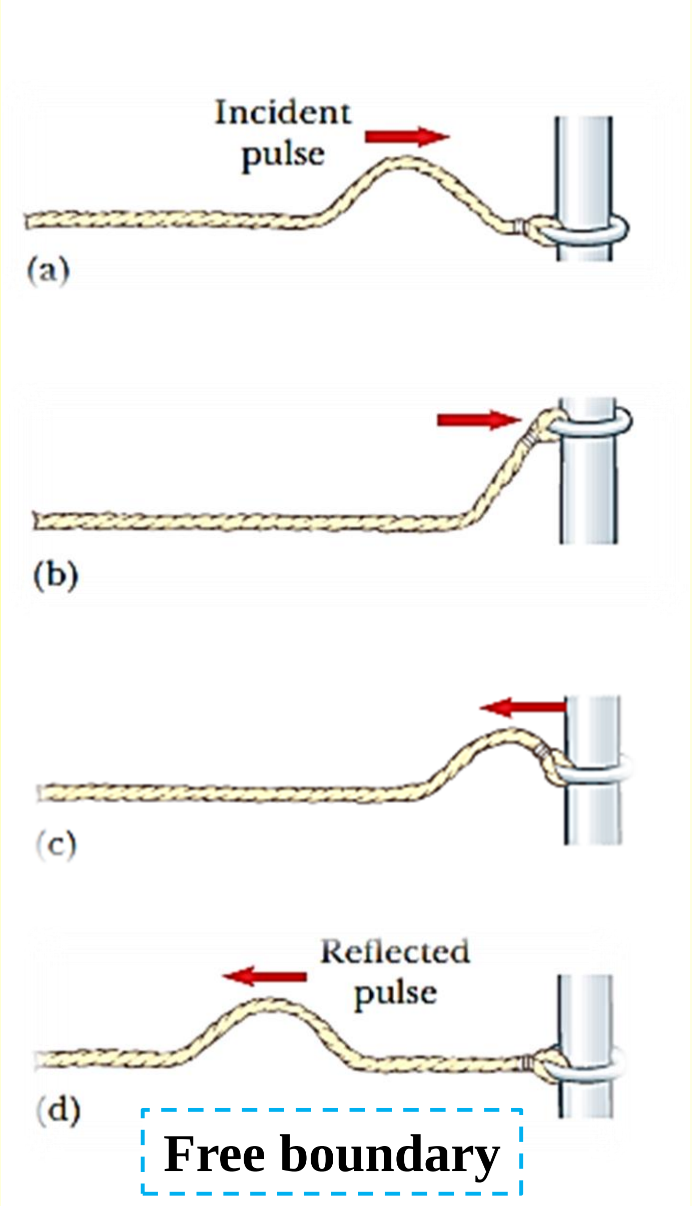




- **1-D waves**
- **2-D waves**
- **3-D waves**

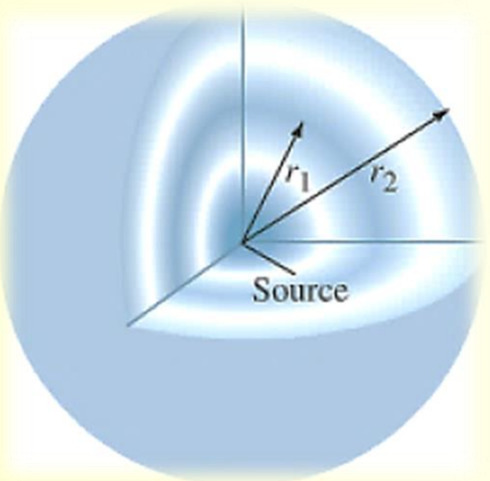
Reflection of Waves





Energy Transported by Waves

$$E = \frac{1}{2}kA^2 \Rightarrow \textbf{wave energy} \propto A^2$$



Intensity *I*

[SI unit: W/m^2]

$$I = \frac{\frac{energy}{area}}{time} = \frac{power}{area}$$

$$E \propto A^2$$

$$\Rightarrow I \propto A^2$$

As for spherical waves (sound waves):

$$I = \frac{power}{area} = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

$$\Rightarrow \boxed{A \propto \frac{1}{r}}$$

Sound Wave

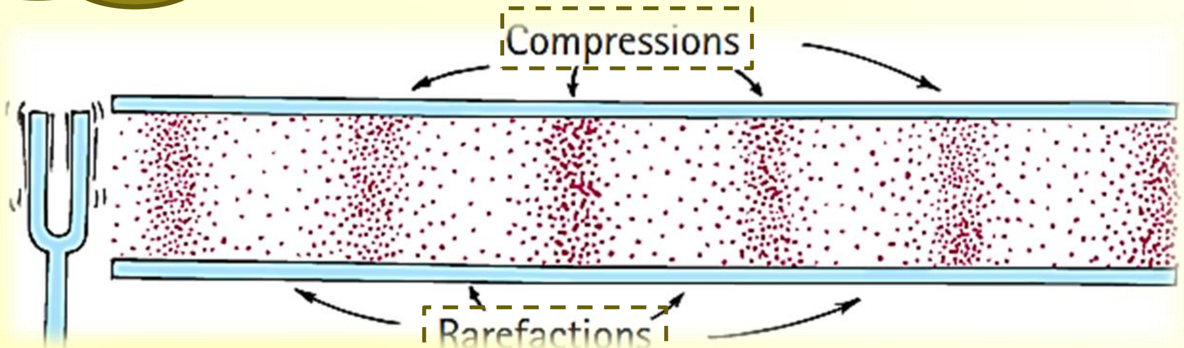
Longitudinal wave

Loudness

(related to *Intensity I*)

Pitch

(related to *Frenquency f*)



Audible frequency range:

20 *Hz* ~ 20000 *Hz*

ultrasonic *vs* **supersonic**

high frequency
(*f* > 20000 *Hz*)

high speed
(*v* > 343 *m/s*)

infrasonic waves

(*f* < 20 *Hz*)

- **Speed of sound:**
  - depends on medium and temperature.
  - $v_{\text{solid}} > v_{\text{liquid}} > v_{\text{gas}}$
  - speed in air:

$$v = (331\text{m/s})\sqrt{\frac{T}{273\text{K}}}$$

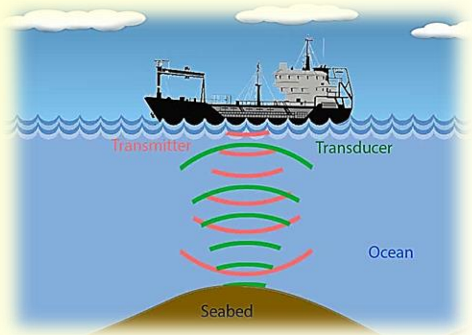
Table 1 Speed of Sound in Various Media

Medium	v (m/s)
Gases	
air (0°C)	331
air (25°C)	346
air (100°C)	366
helium (0°C)	972
hydrogen (0°C)	1290
oxygen (0°C)	317
Liquids at 25°C	
methyl alcohol	1140
sea water	1530
water	1490
Solids	
aluminum	5100
copper	3560
iron	5130
lead	1320
vulcanized rubber	54

Reflection of Sound

echo: ***the reflection of sound***

- **Applications of sonic reflection:**
  - medical imaging --- to remove tumors, break up kidney stones, detect fetal growth
  - bats *or* dolphin emits ultrasound to locate and identify objects in their environment.
  - ultrasonic ranging unit --- to provide an almost instantaneous measurement of the distance between the camera and the object to be photographed.
  - Sonar (sound navigation & ranging)--- to navigate, communicate with or detect objects on or under the surface of the water, e.g. other vessels.



Intensity of Sound: Decibels

Audible intensity range:

10^{-12} W/m^2 ~ 1 W/m^2

Sound level β

$$\beta\,(\mathrm{dB}) = 10\lg\frac{I}{I_0} \quad (I_0 = 1.0\times10^{-12}\ \mathrm{W/m^2},$$

the sound intensity at the threshold of he

Table 2 Conversion of Intensity to Decibel Level

Intensity (W/m^2)	Decibel level (dB)	Examples
1.0×10^{-12}	0	threshold of hearing
1.0×10^{-11}	10	rustling leaves
1.0×10^{-10}	20	quiet whisper
1.0×10^{-9}	30	whisper
1.0×10^{-8}	40	mosquito buzzing
1.0×10^{-7}	50	normal conversation
1.0×10^{-6}	60	air conditioning at 6 m
1.0×10^{-5}	70	vacuum cleaner
1.0×10^{-4}	80	busy traffic, alarm clock
1.0×10^{-3}	90	lawn mower
1.0×10^{-2}	100	subway, power motor
1.0×10^{-1}	110	auto horn at 1 m
1.0×10^{0}	120	threshold of pain
1.0×10^{1}	130	thunderclap, machine gun
1.0×10^{3}	150	nearby jet airplane

Unit: *decibel* (dB) or *bel* (B) $\boxed{\mathbf{1\ B = 10\ dB}}$

When $I = 1.0\times10^{-10}$ W/m^2, $\beta = 10\lg\dfrac{1.0\times10^{-10}}{1.0\times10^{-12}} = 20\ \mathrm{dB}$

When $I = I_0$, $\beta = 10\lg\dfrac{1.0\times10^{-12}}{1.0\times10^{-12}} = 0\ \mathrm{dB}$

0 dB $\neq$ zero intensity

Each 10 dB corresponds to a 10-fold change in I.

【Exercise】

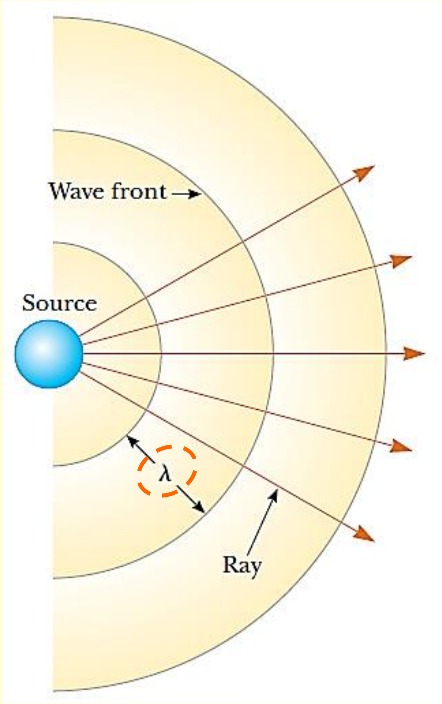
1. A sound wave travels through a metal rod with wavelength λ and frequency f. Which of the following best describes the wave when it passes into the surrounding air?

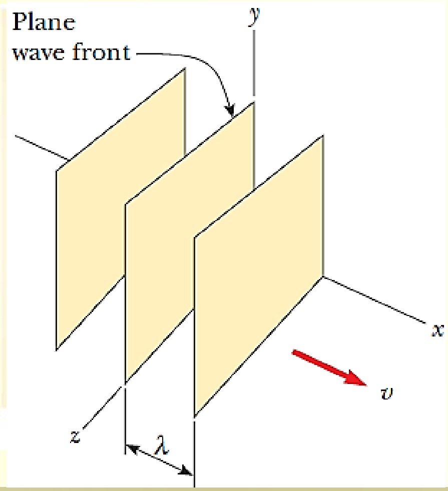
	Wavelength	Frequency
(A)	Less than λ	Equal to f
(B)	Less than λ	Less than f
(C)	Greater than λ	Equal to f
(D)	Greater than λ	Less than f

2. An observer is 2 m from a source of sound waves. By how much will the sound level decrease if the observer moves to a distance of 20 m?
   - (A) 1 dB
   - (B) 2 dB
   - (C) 10 dB
   - (D) 20 dB

Wave Front & rays

- **Wave front:**
  - a series of circular arcs (lines of maximum intensity) to graphically represent spherical waves
  - distance between adjacent wave fronts equals the wavelength λ
- **Ray:**
  - the radial lines pointing outward from the source & perpendicular to wave fronts
- **Plane waves:**
  - at distances from the source that are great relative to λ, the wave front can be approximated with parallel planes, called *plane waves*.

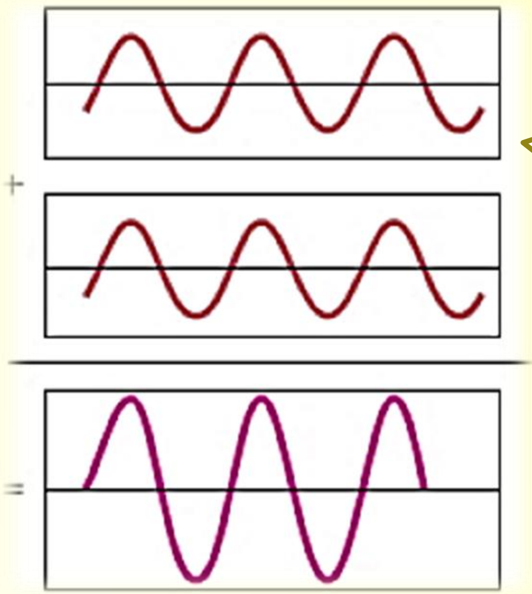




Interference

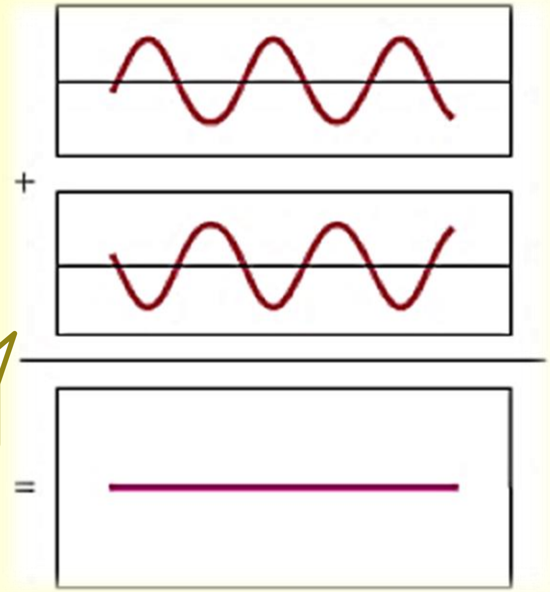
- **Superposition Principle:**

  When two or more traveling waves encounter each other while moving through a medium, the resultant displacement at each point will be *the vector sum* of the displacement of the component waves.



Constructive interference
--- in phase

Destructive interference
--- 180^o out of phase



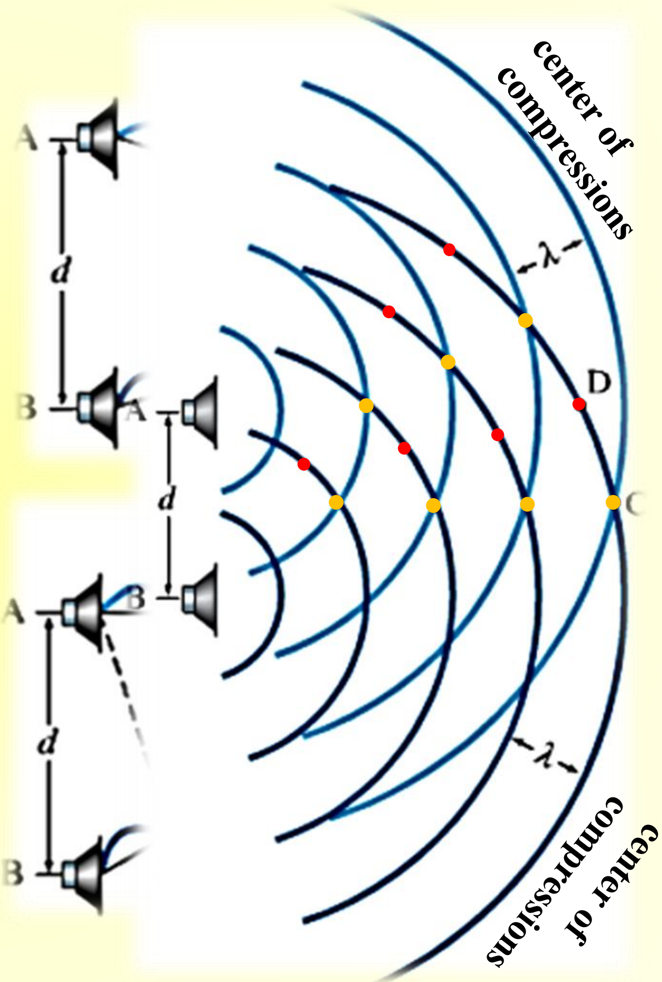
Interference of Sound in Space

C: ***constructive interference***

$$x_{BC} - x_{AC} = n\lambda, n = 0, \pm 1, \pm 2 \cdots$$

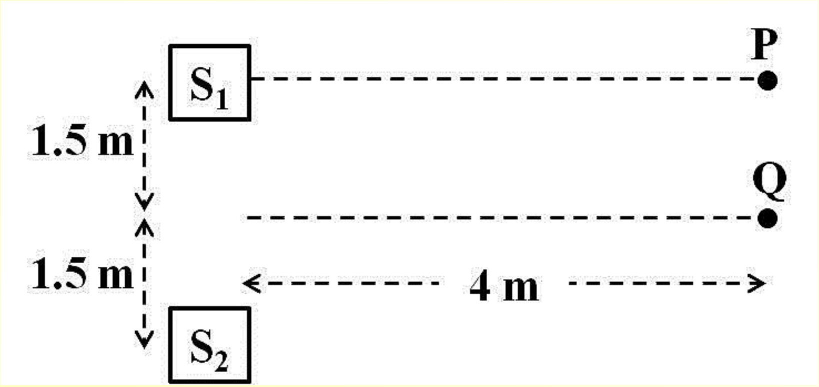
D: ***destructive interference***

$$x_{BD} - x_{AD} = (2n-1)\frac{\lambda}{2}, n = 0, \pm 1, \pm 2 \cdots$$



【Exercise】

In the figure below, two speaker, S_1 and S_2, emit sound waves of wavelength 2 m, in phase with each other. Let A_P be the amplitude of the resultant wave at point P, and A_Q the amplitude of the resultant wave at point Q. How does A_P compare to A_Q ?



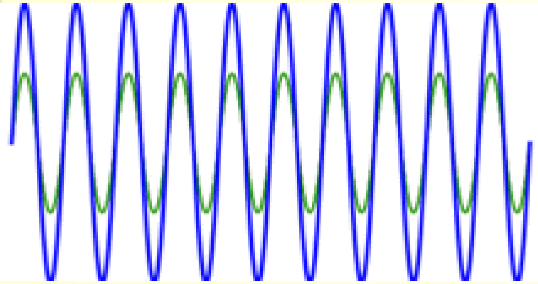
(A) $A_P < A_Q$

(B) $A_P = A_Q$

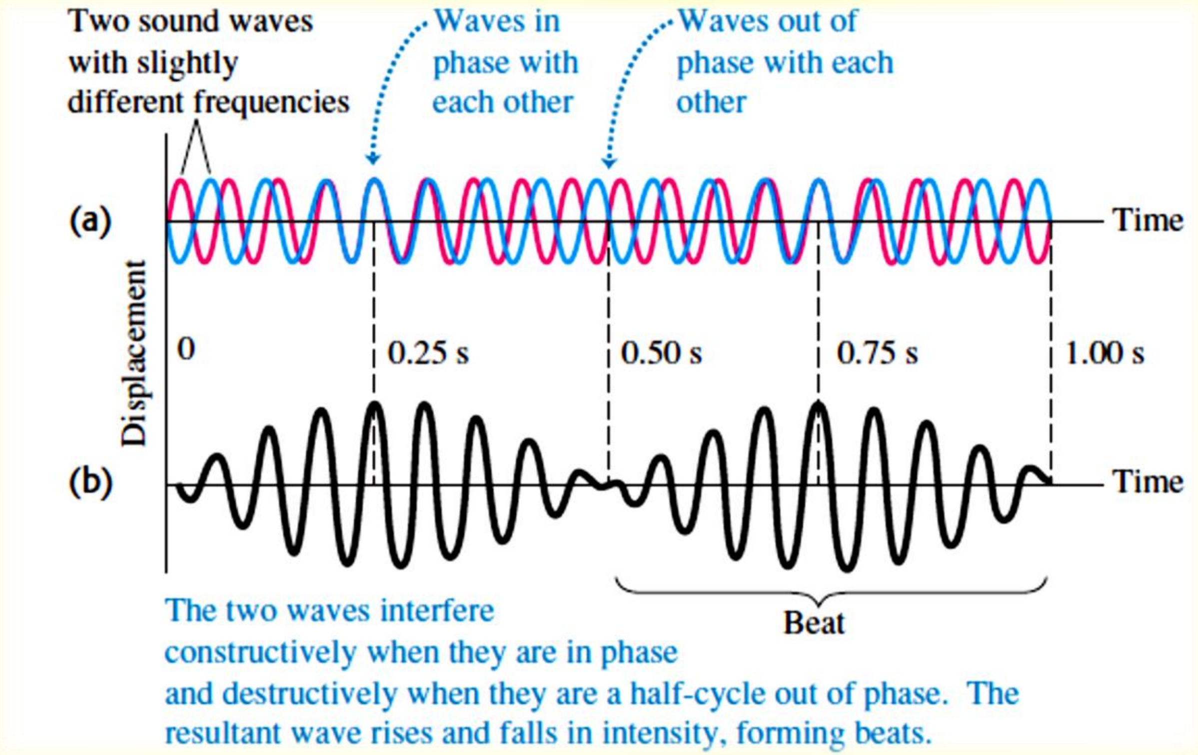
(C) $A_P > A_Q$

(D) $A_P < 0, A_Q > 0$

Interference of Sound in Time -- Beats



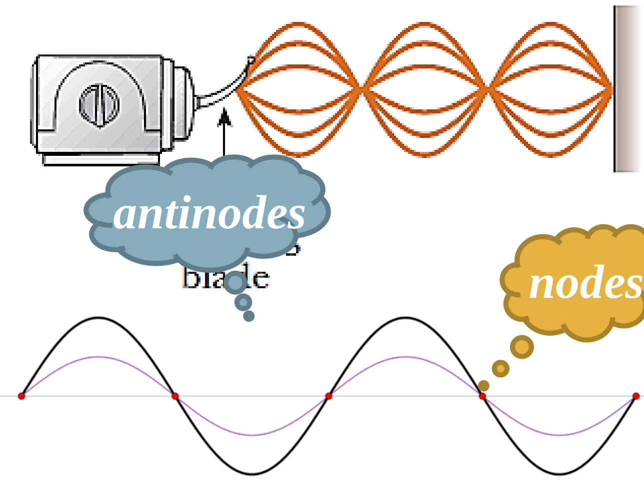
- A beat is the sum of two propagating waves (*same amplitude & similar frequencies*) traveling in the same direction.
- Application: sensitive method for comparing frequencies, *e.g. tuning a piano.*



$$f_{beat} = \left| f_A - f_B \right|$$

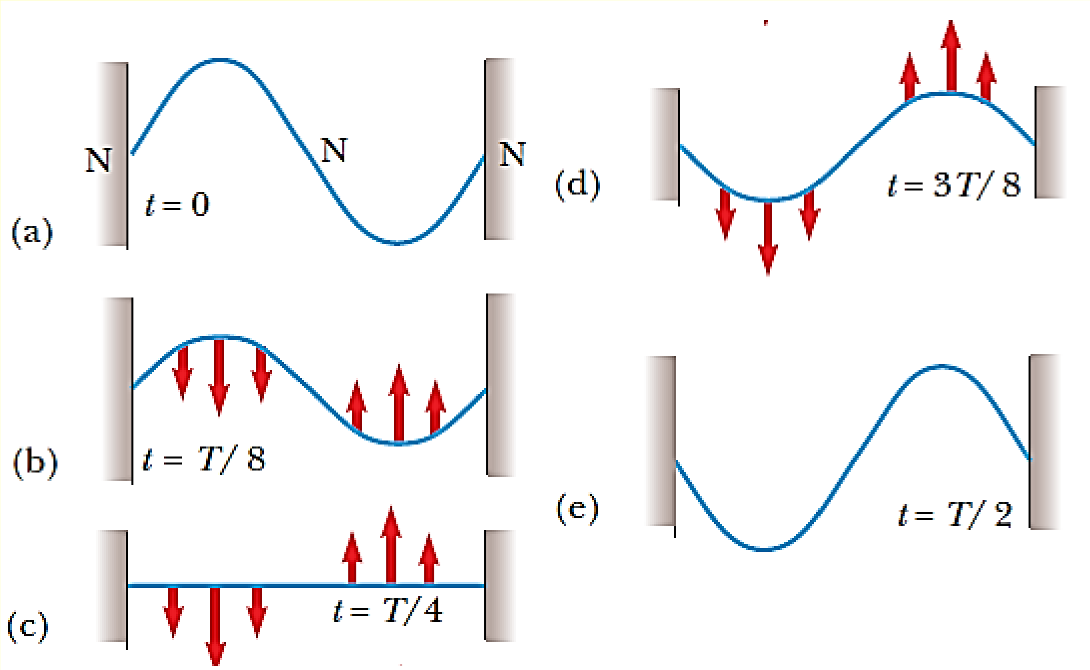
Standing Waves (Stationary Waves)

- A standing wave is a wave which oscillates in time but whose peak amplitude profile does not move in space.



- The locations at which the amplitude is minimum are called *nodes*, and the locations where the amplitude is maximum are called *antinodes*.

- All points on the string oscillate together vertically with the same frequency, but different points have different amplitudes of motion.



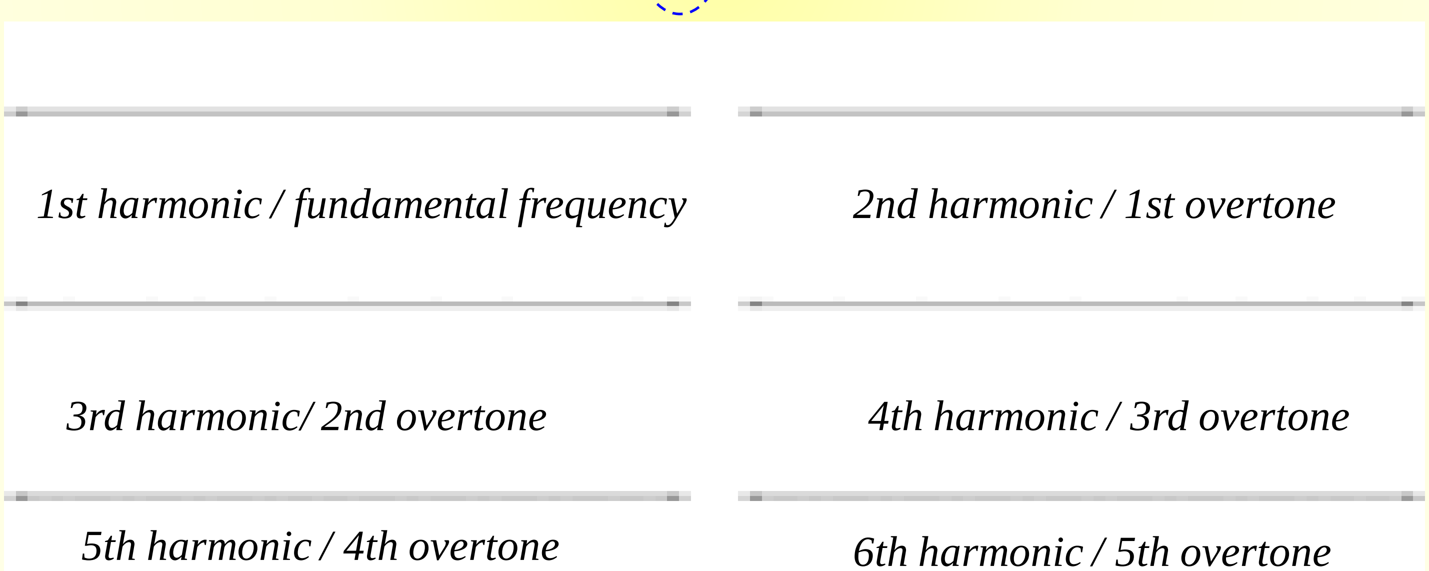
Harmonics

➢ Harmonics in a string:

- All the standing-wave frequencies in a string are called ***harmonics***.

$$L = \frac{n\lambda_n}{2} \quad (n = 1,2,3\cdots)$$

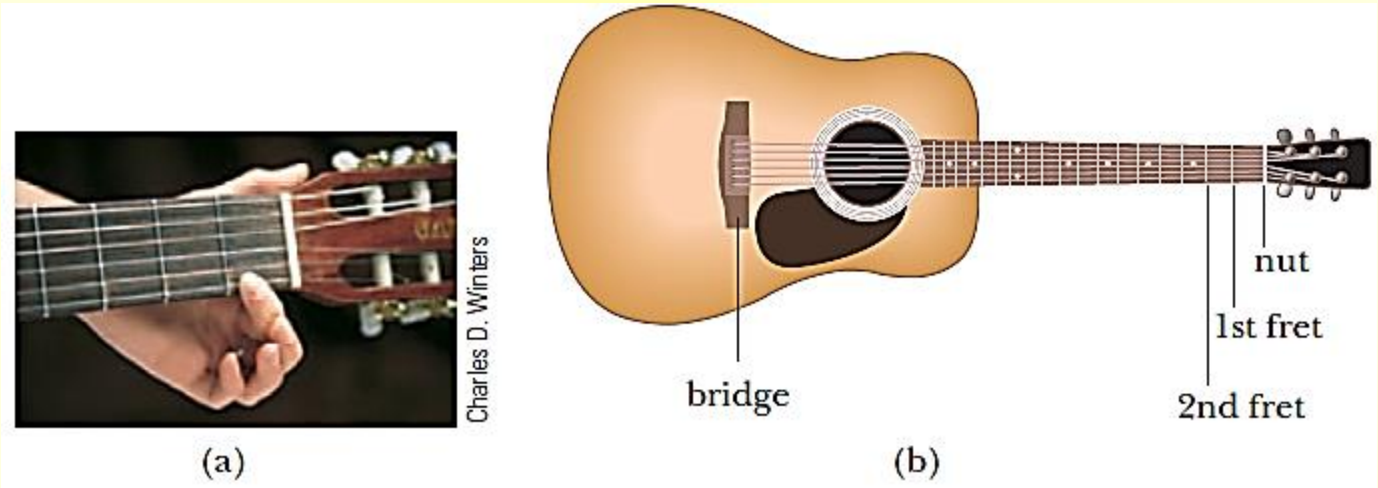
$$f_n = \frac{v}{\lambda_n} = n\underbrace{\frac{v}{2L}}_{=f_1} \quad (n = 1,2,3\cdots)$$



【Exercise】

The high E string on a certain guitar measures 64.0 cm in length and has a fundamental frequency of 329 Hz. When a guitarist presses down so that the string is in contact with the first fret, the string is shortened so that it plays an F note that has a frequency of 349 Hz.

(a) How far is the fret from the nut?



(a) Find the distance from the nut to the first fret:

$$f_1 = \frac{v}{2L_0} \Rightarrow v = 2L_0 f_1 = 421\,\mathrm{m/s} \Rightarrow L = \frac{v}{2f} = 60.3\,\mathrm{cm} \Rightarrow \Delta x = L_0 - L = 3.7\,\mathrm{cm}$$

The high E string on a certain guitar measures 64.0 cm in length and has a fundamental frequency of 329 Hz.

(b) Overtones can be produced on a guitar string by gently placing the index finger in the location of a node of a higher harmonic. The fundamental frequency is thereby suppressed, making it possible to hear overtones. Where on the guitar string relative to the nut should the finger be lightly placed so as to hear the second harmonic? The fourth harmonic?

(b) The second harmonic has a wavelength $\lambda_2 = L_0 = 64.0$ cm.

The distance from nut to node corresponds to half a wavelength:

$$\Delta x = \frac{1}{2}\lambda_2 = 32.0\text{cm}$$

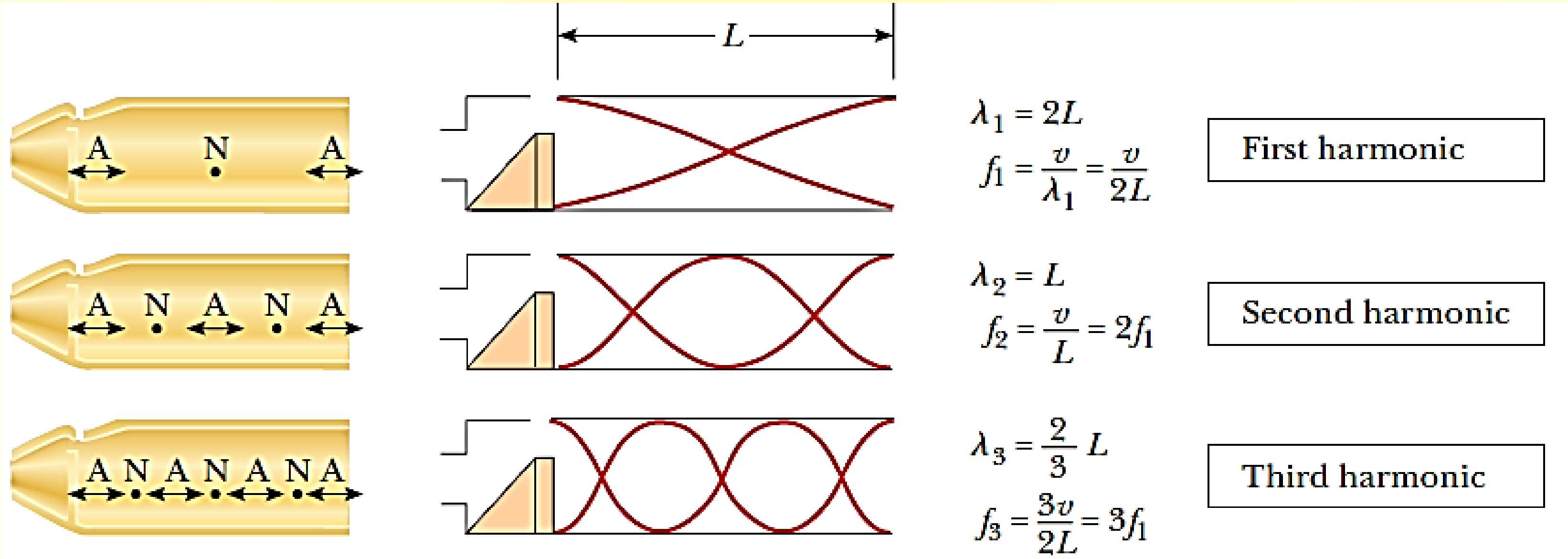
The fourth harmonic, of wavelength $\lambda_4 = \frac{1}{2}L_0 = 32.0\text{cm}$, has three nodes between the endpoints:

$$\Delta x = \frac{1}{2}\lambda_4 = 16.0\text{cm},\ \ \Delta x = 2(\frac{\lambda_4}{2}) = 32.0\text{cm},\ \ \Delta x = 3(\frac{\lambda_4}{2}) = 48.0\text{cm}$$

Harmonics in an air column:

- If one end is closed, a node must exist at that end because the movement of air is restricted. If the end is open, the elements of air have complete freedom of motion, and an antinode exists.

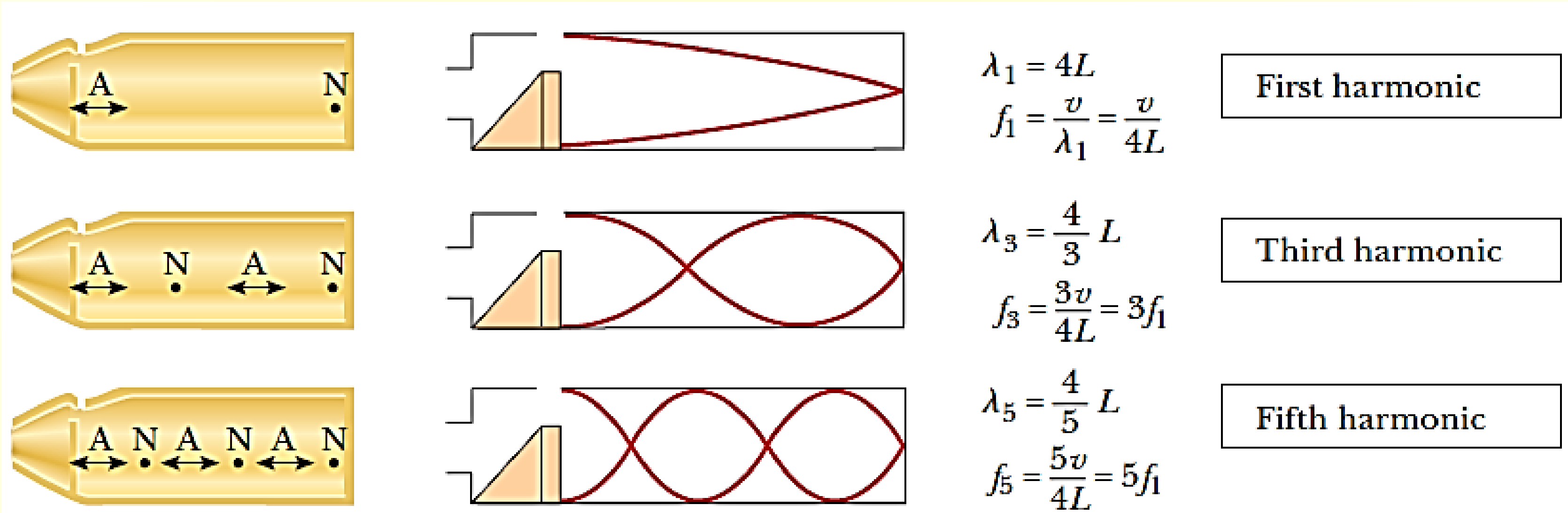
✓ *Open at both ends:*



$$f_n = n\frac{v}{2L}\ (n = 1,2,3\cdots)$$

All harmonics are present!

✓ *Closed at one end, open at the other:*



$$f_n = n\frac{v}{4L} (n = 1,3,5\cdots)$$

Only odd harmonics are present!

【Exercise】

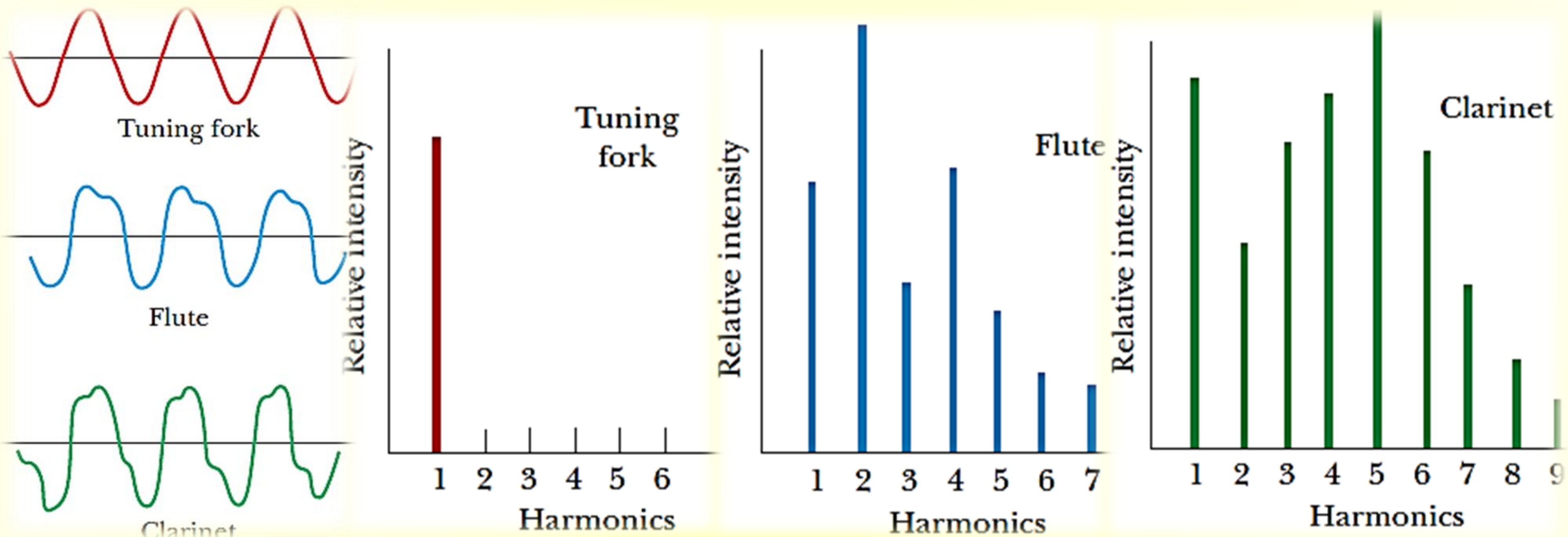
1. A pipe open at both ends resonates at a fundamental frequency f_{open}. When one end is covered and the pipe is again made to resonate, the fundamental frequency is f_{closed}. Which of the following expressions describes how these two resonant frequencies compare?

   (a) $f_{closed} = f_{open}$ (b) $f_{closed} = \frac{3}{2} f_{open}$ (c) $f_{closed} = 2f_{open}$

   (d) $f_{closed} = \frac{1}{2} f_{open}$ (e) none of these

2. Balboa Park in San Diego has an outdoor organ. When the air temperature increases, the fundamental frequency of one of the organ pipes (a) increases (b) decreases (c) stays the same (d) impossible to determine. (The thermal expansion of the pipe is negligible.)

Quality/Timbre of Sound (Tone Color)

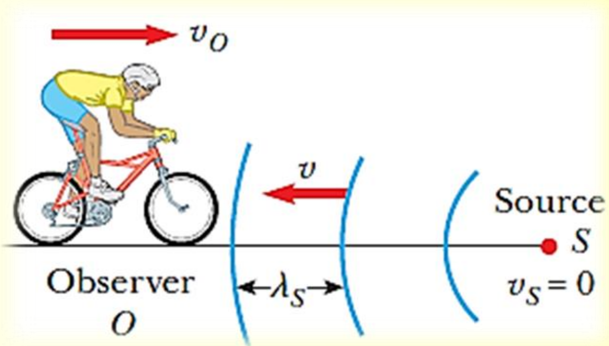


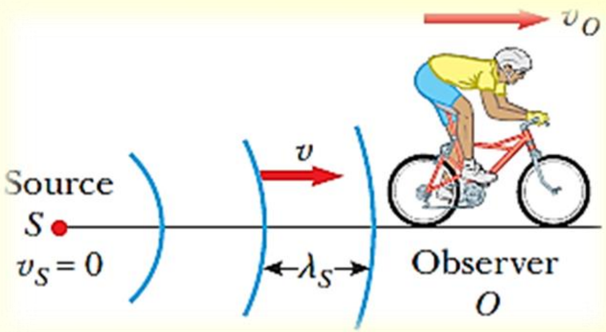
- The quality depends on the mixture of harmonics in the sound --- their number and their relative amplitudes.
- The presence of several harmonics on a musical instrument gives it characteristic sound, which enables us to distinguish one from another even when they are producing identical fundamental frequencies.

Doppler Effect

- The Doppler effect is the **change in frequency** of a wave in relation to observer who is moving relative to the wave source.

➢ **Case 1: the observer is moving relative to a stationary source**





- When the observer is moving *toward* the source,

  During a time interval t, additional wave fronts detected $= \dfrac{v_O t}{\lambda_S}$

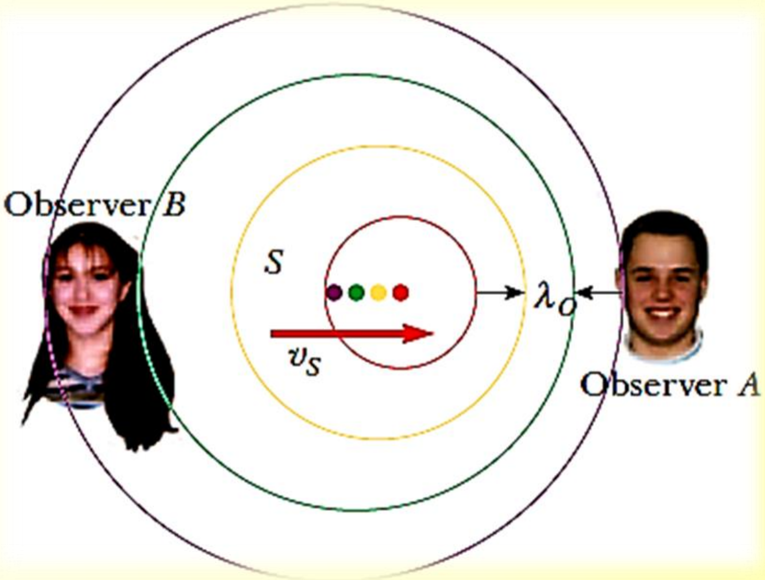
  Thus, the frequency heard by the observer:

$$\left.\begin{aligned} f_O &= f_S + \frac{v_O}{\lambda_S} \\ \lambda_S &= \frac{v}{f_S} \end{aligned}\right\} \Rightarrow \boxed{f_O = f_S\left(\frac{v + v_O}{v}\right)}$$

- When the observer moves *away from* the source,

$$\boxed{f_O = f_S\left(\frac{v - v_O}{v}\right)}$$

➢ Case 2: the source is moving relative to a stationary observer



- When the source is moving *toward* a stationary observer,

  During a period T, the source moves a distance $v_S T = \frac{v_S}{f_S}$

  Thus, the observed wavelength:

$$\left.\begin{aligned} \lambda_O &= \lambda_S - \frac{v_S}{f_S} \\ \lambda_S &= \frac{v}{f_S} \\ f_O &= \frac{v}{\lambda_O} \end{aligned}\right\} \Rightarrow \boxed{f_O = f_S\left(\frac{v}{v - v_S}\right)}$$

- When the source is moving *away from* a stationary observer,

$$\boxed{f_O = f_S\left(\frac{v}{v + v_S}\right)}$$

➢ General Case

$$f_O = f_S\left(\frac{v+v_O}{v-v_S}\right) \Leftrightarrow \frac{f_O}{v+v_O} = \frac{f_S}{v-v_S}$$

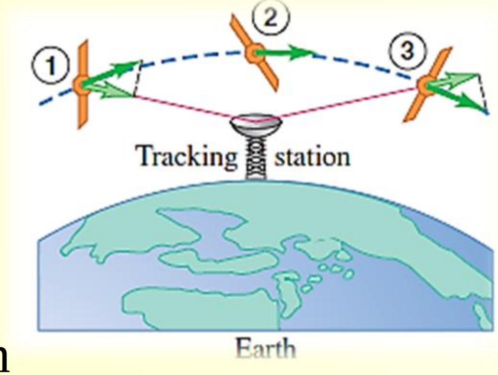
- When the observer moves *toward* the source, a *positive* speed is substituted for v_O; when the observer moves *away from* the source, a *negative* speed is substituted for v_O. Similarly, a *positive* speed is substituted for v_S when the source moves *toward* the observer, a *negative* speed when the source moves *away from* the observer.

- The rules to be obeyed: The word *toward* is associated with an *increase* in the observed frequency; the words *away from* are associated with a *decrease* in the observed frequency.

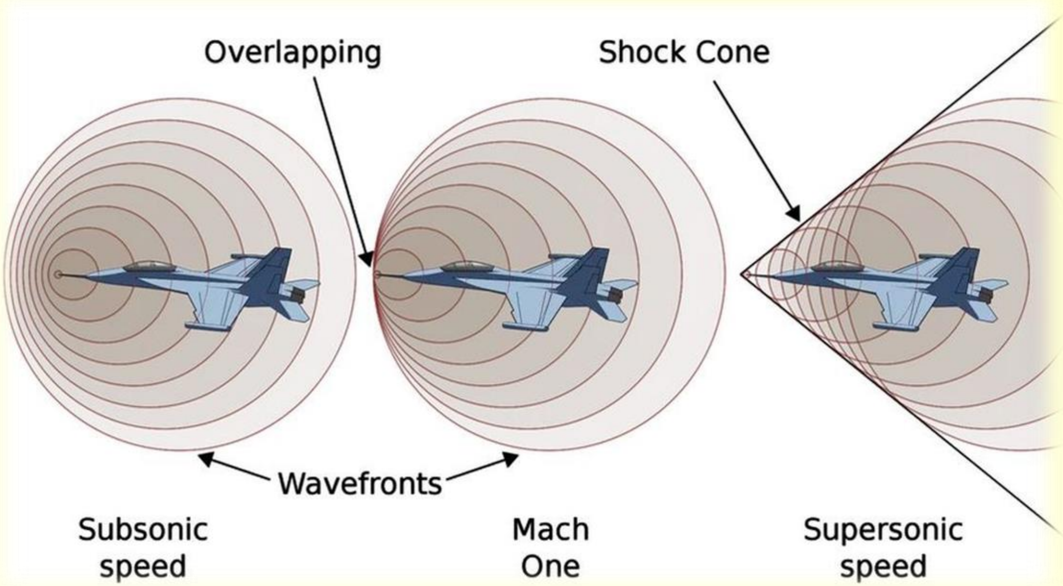
➢ Applications of Doppler Effect for radio waves:

✓ *Used to measure wind velocities in the atmosphere*

✓ *Used to track satellites and other space vehicles.*



- The conical wave front produced when $v_S > v$ is known as a shock wave.



The first supersonic airplane, the Bell X-1, with Mach 1.06 on Oct. 14, 1947

【Exercise】

A source of 4 kHz sound waves travels at 1/9 the speed of sound toward a detector that is moving at 1/9 the speed of sound toward the source.

(a) what is the frequency of the waves as they are received by the detector?

(b) how does the wavelength of the detected waves compare to the wavelength of the emitted waves?

(a) The frequency of the detected waves: $f_O = f_S(\frac{v+v_O}{v-v_S}) = 5\text{kHz}$

(b) The wavelength of the detected waves:

$$\lambda_O = \lambda_S - \frac{v_S}{f_S} = \frac{v-v_S}{f_S} = \frac{8v}{9f_S}$$

The wavelength of the emitted waves:

$$\lambda_S = \frac{v}{f_S}$$

$$\Rightarrow \frac{\lambda_O}{\lambda_S} = \frac{8}{9}$$