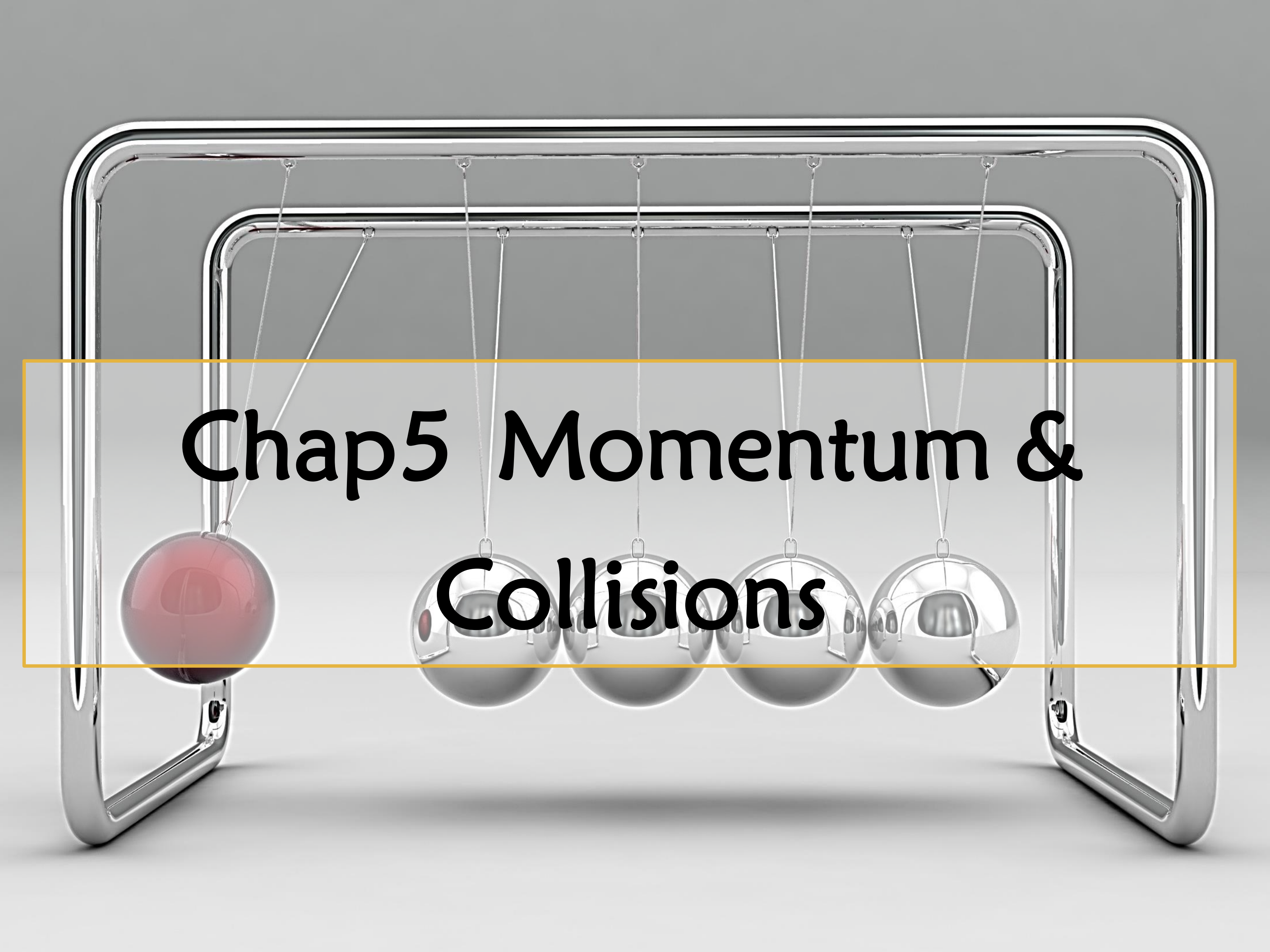
Chap5 Momentum & Collisions



Linear Momentum

vector

- $\vec{p} = m\vec{v}$ ➜ $\begin{cases} p_x = mv_x \\ p_y = mv_y \end{cases}$ **SI unit: kg·m/s**

- $\vec{F}_{net} = \Sigma\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$ **Newton's Second Law**

$$= \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$= m\vec{a}$$

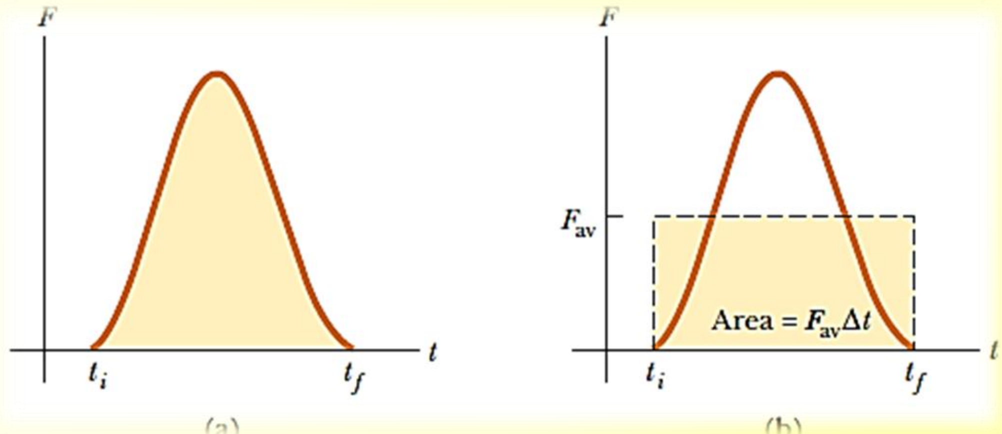
Impulse

vector

- $\vec{I} = \vec{F}\Delta t$ **SI unit: N·s = kg·m/s**

➢ $\vec{F}_{net}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$

Impulse-Momentum Theorem



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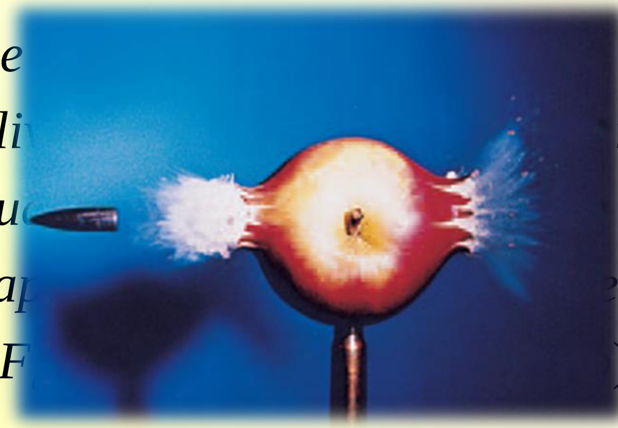


Test your Understanding of Momentum

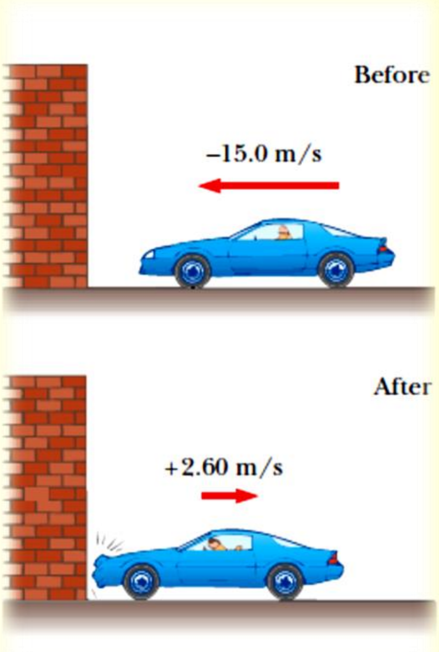
Two objects with masses m_1 and m_2 ($m_1 < m_2$) have equal kinetic energy. How do the magnitudes of their momenta compare?

(A) $p_1 < p_2$ ✓ **(B) $p_1 = p_2$** **(C) $p_1 > p_2$**

(D) Not enough information is given

【Example】 "How good are the bumpers?"

In a crash test, a car of mass 1.50×10^3 kg collides with a wall and rebounds. The initial and final velocities of the car are v_i = -15.0 m/s and v_f = 2.60 m/s, respectively. If the collision lasts for 0.150 s, find (a) the impulse delivered to the car due to the collision and (b) the size and direction of the average force exerted on the car.



(a)

(b)

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(a) Calculate the initial and final momenta of the car:

$$p_i = mv_i = -2.25 \times 10^4 \,\mathrm{kg \cdot m/s}$$

$$p_f = mv_f = +0.390 \times 10^4 \,\mathrm{kg \cdot m/s}$$

The impulse is just the difference between the final and initial momenta:

$$I = p_f - p_i = 2.64 \times 10^4 \,\mathrm{kg \cdot m/s}$$

(b) According to the impulse-momentum theorem, the average force exerted on the car is:

$$F_{av} = \frac{I}{\Delta t} = +1.76 \times 10^5 \,\mathrm{N}$$

Conservation of Momentum

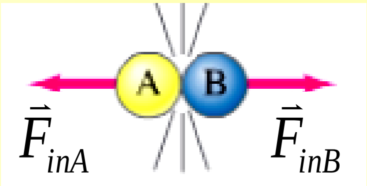
(*for a isolated system*)

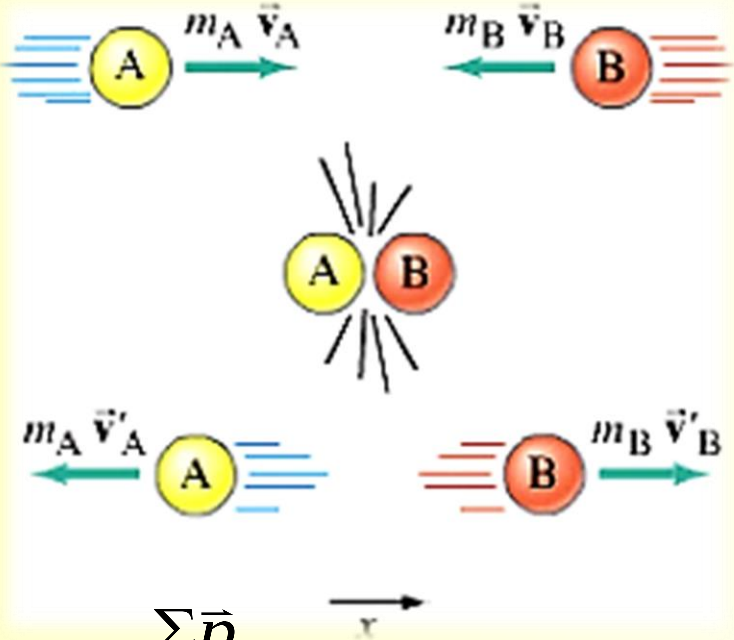
external force $\Sigma\vec{F}_{ex} = 0$

internal force $\vec{F}_{in} \neq 0$

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

- *Two-object system*



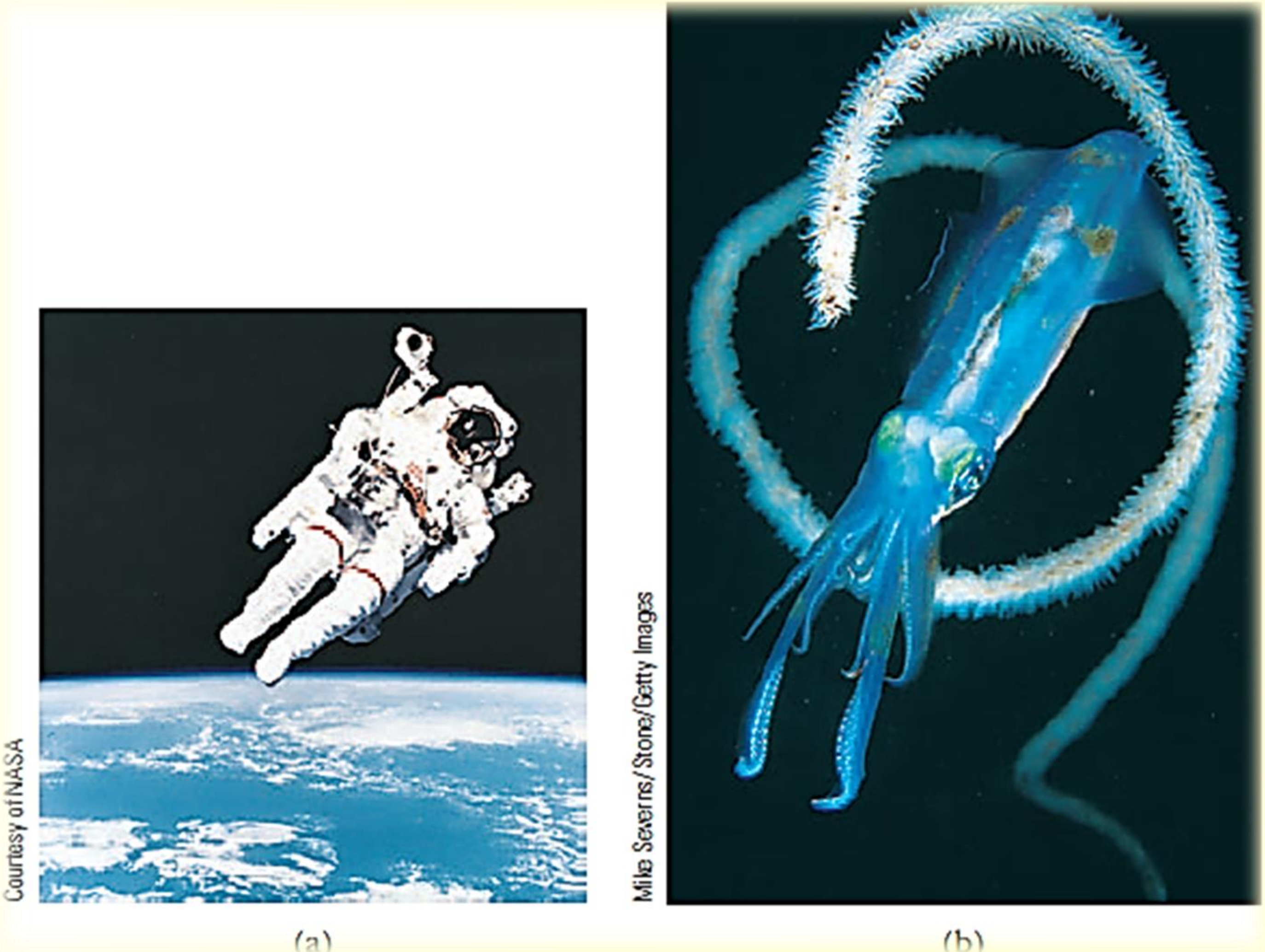


$$\Delta\vec{p}_A = \vec{F}_{inA}\Delta t,\ \ \Delta\vec{p}_B = \vec{F}_{inB}\Delta t$$

$$\left.\begin{aligned} m_A\vec{v}'_A - m_A\vec{v}_A &= \vec{F}_{inA}\Delta t \\ m_B\vec{v}'_B - m_B\vec{v}_B &= \vec{F}_{inB}\Delta t \\ \vec{F}_{inB} &= -\vec{F}_{inA} \end{aligned}\right\} \Rightarrow \underbrace{m_A\vec{v}_A + m_B\vec{v}_B}_{\Sigma\vec{p}_i} = \underbrace{m_A\vec{v}'_A + m_B\vec{v}'_B}_{\Sigma\vec{p}_f}$$

head-on collision:

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$



Conservation of momentum is the principle behind these two propulsion systems. (*a*) *The force from a nitrogen-propelled, hand-controlled device allows an astronaut to move about freely in space without restrictive tethers.* (*b*) *A squid propels itself by expelling water at high velocity.*

Recoil

【Example】 The Archer

An archer stands at rest on frictionless ice and fires 0.5-kg arrow horizontally at 50.0 m/s. The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

Let v_{1f} be the archer's velocity and v_{2f} the arrow's velocity.

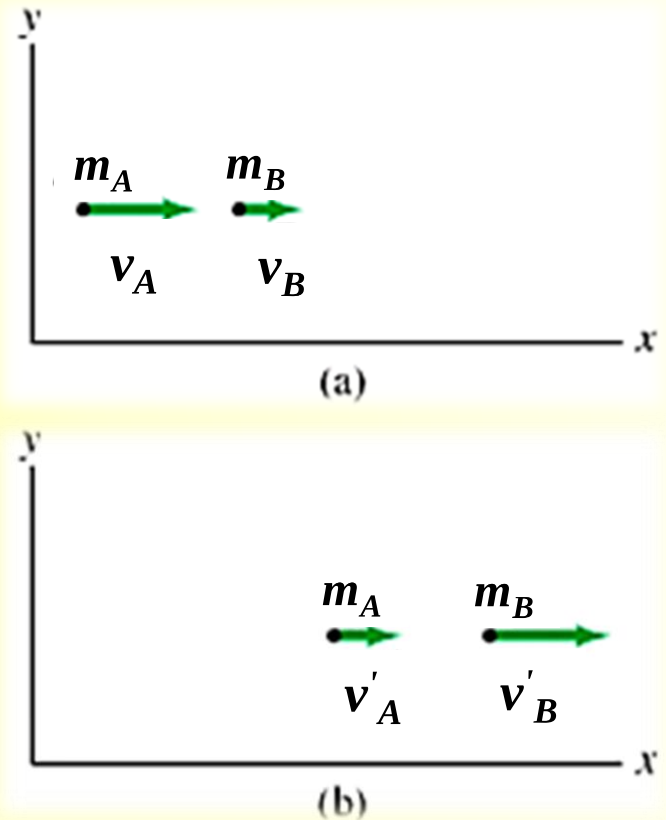
According to the conservation of momentum equation,

$$p_i = p_f \Rightarrow 0 = m_1 v_{1f} + m_2 v_{2f} \Rightarrow v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5\text{kg}}{60.0\text{kg}} \times 50.0\,\text{m/s}$$

$$= -0.42\,\text{m/s}$$

Energy & Momentum in Collision

collision
- **elastic collision**
- **inelastic collision**



Elastic Collision in 1-D

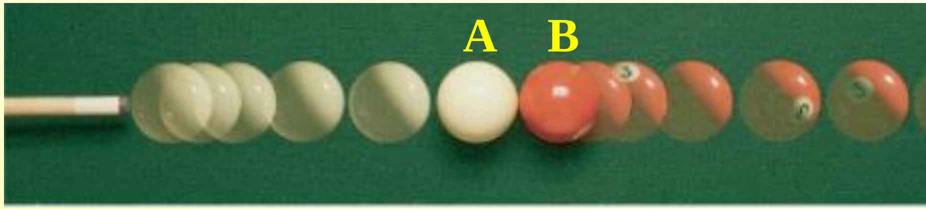
(head-on elastic collision)

$$\begin{cases} m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \\ \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v'^2_A + \frac{1}{2} m_B v'^2_B \end{cases}$$

$$\boxed{v_A - v_B = v'_B - v'_A}$$

【Example】 Collision of Billiard Balls

Billiard ball A of mass *m* moving with speed *v* collides *head-on* with ball B of equal mass at rest. What are the speeds of the two balls after the collision, assuming it is elastic?



$$m_A = m_B = m,\ \ v_A = v,\ \ v_B = 0$$

According to the conservation of momentum equation,

$$mv + 0 = m(v'_A + v'_B) \Rightarrow v = v'_A + v'_B$$

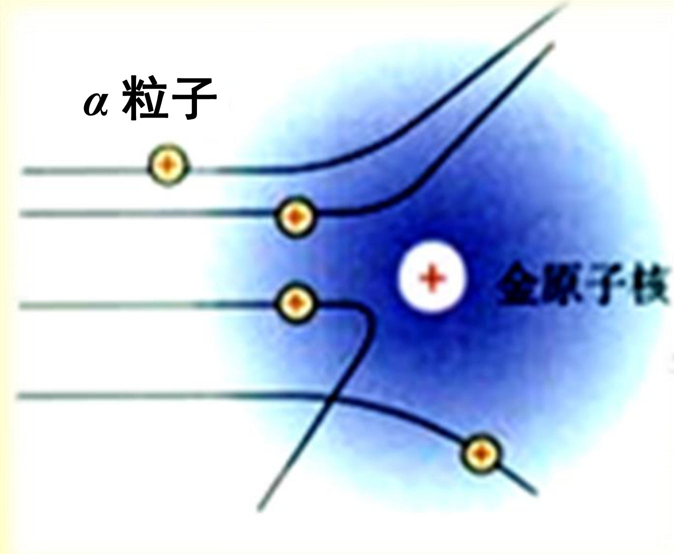
The collision is elastic, and thus

$$v_A - v_B = v'_B - v'_A \Rightarrow v = v'_B - v'_A$$

$$\Rightarrow \begin{cases} v'_A = 0 \\ v'_B = v \end{cases}$$

【Example】 A nuclear collision

A proton (p) of mass 1.01 u (1 u = 1.66 ×10^{-27} kg) traveling with a speed of 3.60 ×10^{4} m/s has an elastic *head-on* collision with He nucleus (m_{He} = 4.00 u) initially at rest. What are the velocities of the proton and He nucleus after the collision?



According to the conservation of momentum equation,

$$m_p v_p + 0 = m_p v'_p + m_{He} v'_{He} \quad ①$$

The collision is elastic, so

$$v_p - 0 = v'_{He} - v'_p$$

⇩

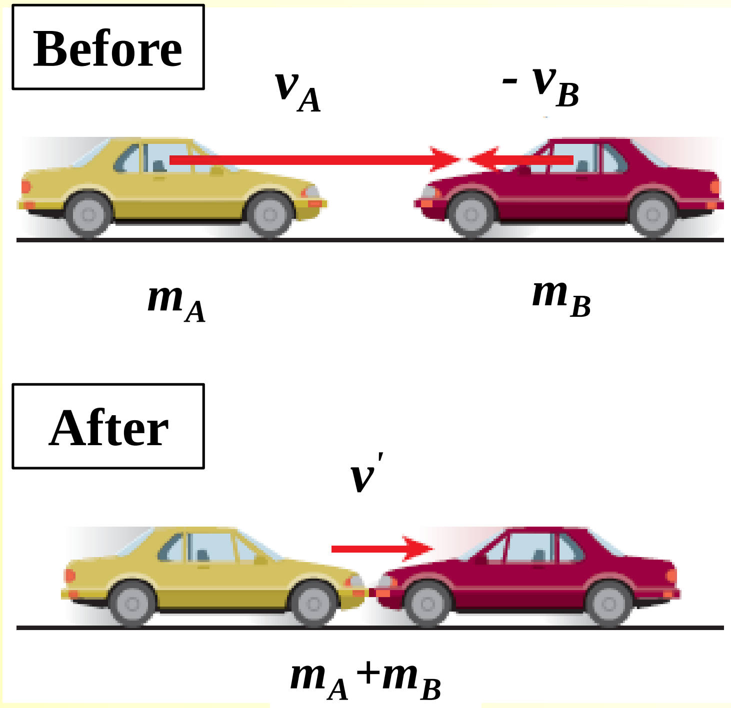
$$v'_p = v'_{He} - v_p \quad ②$$

⇨

$$v'_{He} = \frac{2m_p v_p}{m_p + m_{He}} = 1.45\times10^4\ \text{m/s}$$

$$v'_p = \frac{(m_p - m_{He})v_p}{m_p + m_{He}} = -2.15\times10^4\ \text{m/s}$$

Perfectly Inelastic Collision in 1-D



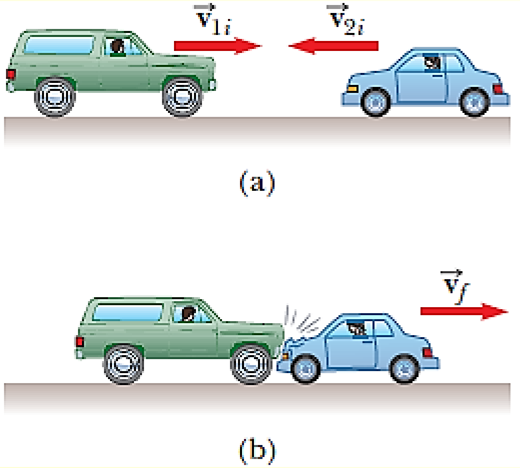
$$m_A v_A + m_B(-v_B) = (m_A + m_B)v'$$

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - K_{lost} = \frac{1}{2}(m_A + m_B)v'^2$$

$$K_{lost} = (\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2) - \frac{1}{2}(m_A + m_B)v'^2$$

【Example】 An SUV Versus a Compact

An SUV with mass 1.80×10^3 kg is traveling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is traveling westbound at -15.0 m/s. The cars collide *head-on*, becoming entangled. (a) Find the speed of the entangled cars after the collision. (b) Find the change in the velocity of each car. (c) Find the change in the kinetic energy of the system consisting of both cars.



(a) According to the conservation of momentum equation,

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \Rightarrow v_f = +5.00\,\mathrm{m/s}$$

(b) Change in velocity of the SUV: $\Delta v_1 = v_f - v_{1i} = -10.0\,\mathrm{m/s}$

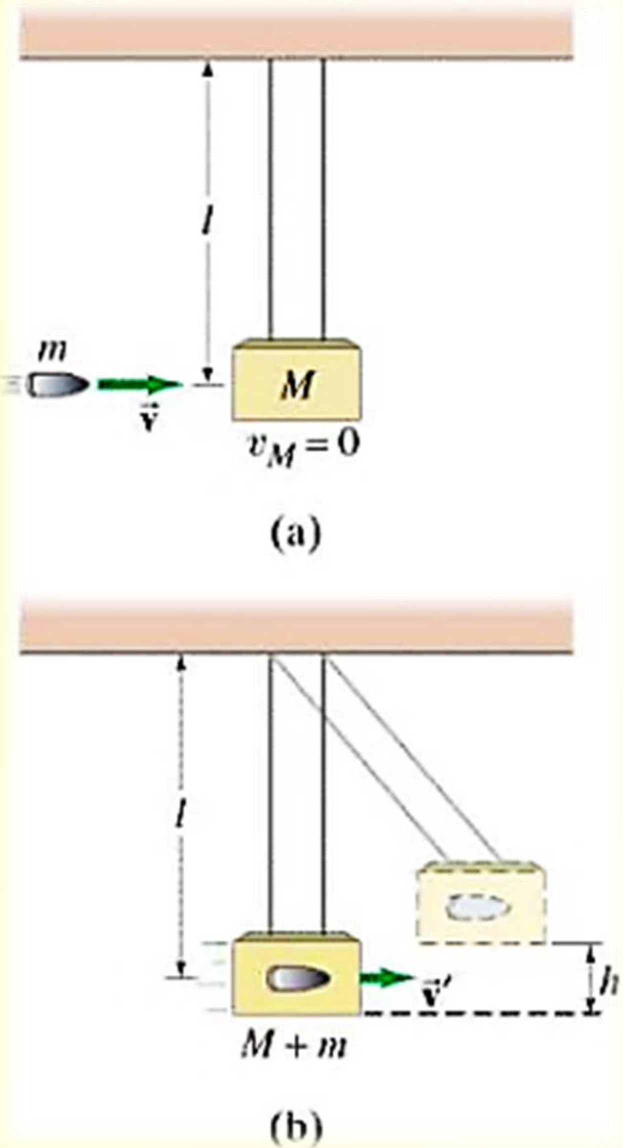
Change in velocity of the compact car: $\Delta v_2 = v_f - v_{2i} = 20.0\,\mathrm{m/s}$

(c) The change in kinetic energy of the system is:

$$\Delta K = K_f - K_i = \frac{1}{2}(m_1 + m_2) v_f^2 - \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2\right) = -2.70 \times 10^5\,\mathrm{J}$$

【Example】 Ballistic pendulum

The ballistic pendulum is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass *m*, is fired into a large block of mass *M*, which is suspended like a pendulum. (usually, *M*>*m*). As a result of the collision , the pendulum and projectile together swing up to a maximum height *h*. **Determine** the relationship between the initial horizontal speed of the projectile, *v*, and the maximum height *h*.



Use conservation of momentum,

$$mv = (m+M)v'$$

$$\Downarrow$$

$$v = \frac{(m+M)v'}{m} \quad ①$$

Apply conservation of energy to the block-bullet system after the collision:

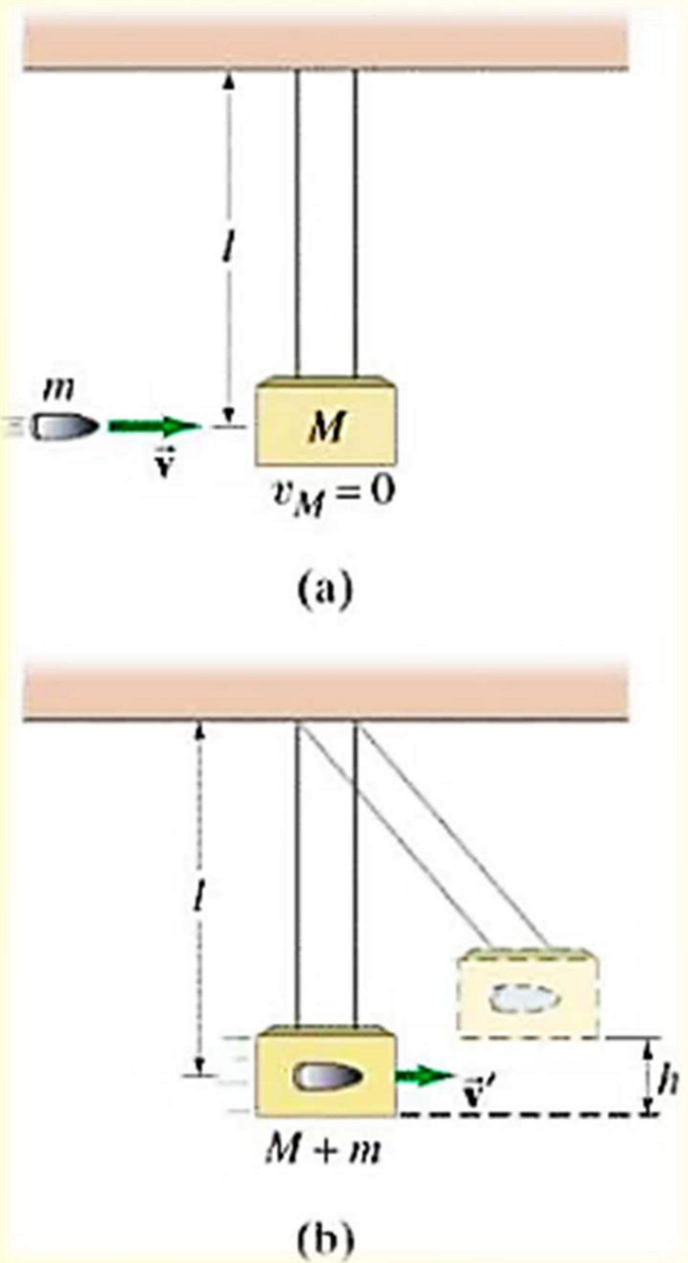
$$\frac{1}{2}(m+M)v'^2 = (m+M)gh$$

$$\Downarrow$$

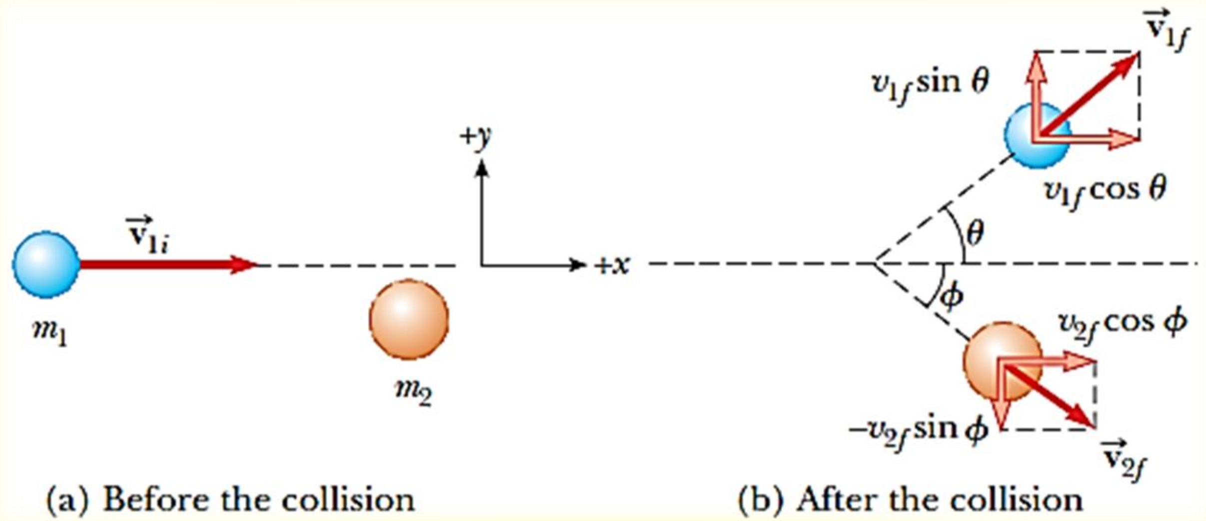
$$v' = \sqrt{2gh} \quad ②$$

According to the equations ① and ②,

$$\boxed{v = \frac{(m+M)}{m}\sqrt{2gh}}$$



Glancing Collisions



- ***x*-component:** $m_1 v_{1i} + 0 = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$
- ***y*-component:** $0 + 0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$

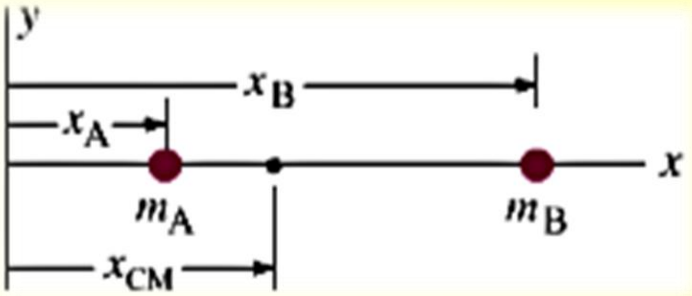
- ***If the collision is elastic,***

$$\frac{1}{2}m_1 v_{1i}^2 + 0 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

Center of Mass (CM)

- The center of mass (CM) and center of gravity (CG) of an object are exactly the same when g doesn't vary significantly over the object.

➢ Location of the CM



$$(m_A + m_B)x_{CM} = m_A x_A + m_B x_B$$

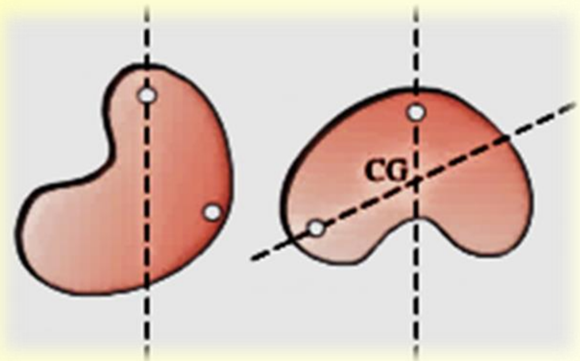
$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M}$$

- *For many particles,*

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + \ldots}{m_A + m_B + m_C + \ldots}$$

$$F_{net} = M \cdot a_{CM}$$

➢ Experimentally finding the location of the CG (CM)



【Example】

An ice-skater is moving at a constant velocity across an icy pond. The skater throws a snowball directly ahead. Which of the following correctly describes the velocity of the center of mass of the skater-snowball system immediately after the snowball is thrown? Assume friction and air resistance are negligible.

(A) It is equal to the velocity of the snowball.

(B) It is equal to the new velocity of the skater.

(C) It is equal to half the original velocity of the skater.

(D) It is equal to the original velocity of the skater.

【Example】

A train is traveling east with constant speed v_t. Two identical spheres are rolling on the floor of one train car. In the frame of reference of the train, the spheres are moving directly toward each other at one instant with the same speed v_p parallel to the train's motion, as shown in Figure. What is the velocity of the center of mass of the spheres in the frame of reference of the train and in the frame of reference of a person standing at rest alongside the train?

	Train	Person
(A)	Zero	Zero
(B) ✓	Zero	v_t east
(C)	v_t east	$v_p + v_t$ east
(D)	$v_p + v_t$ east	Zero