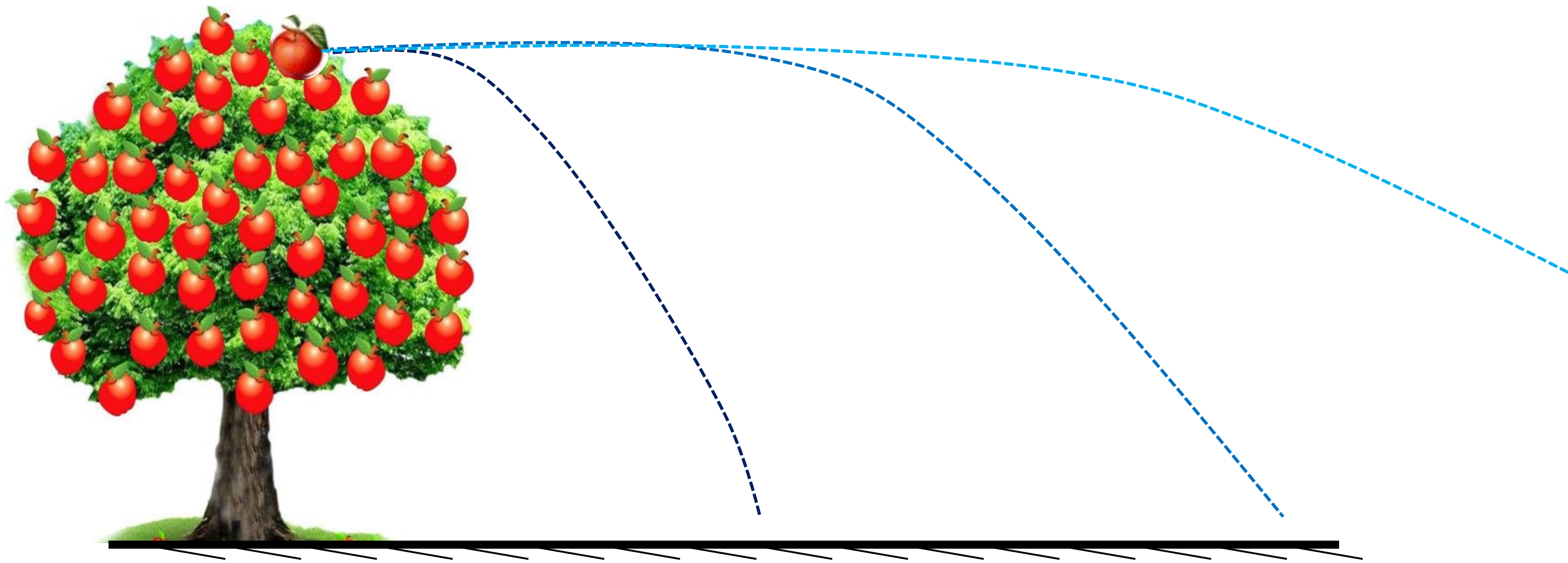
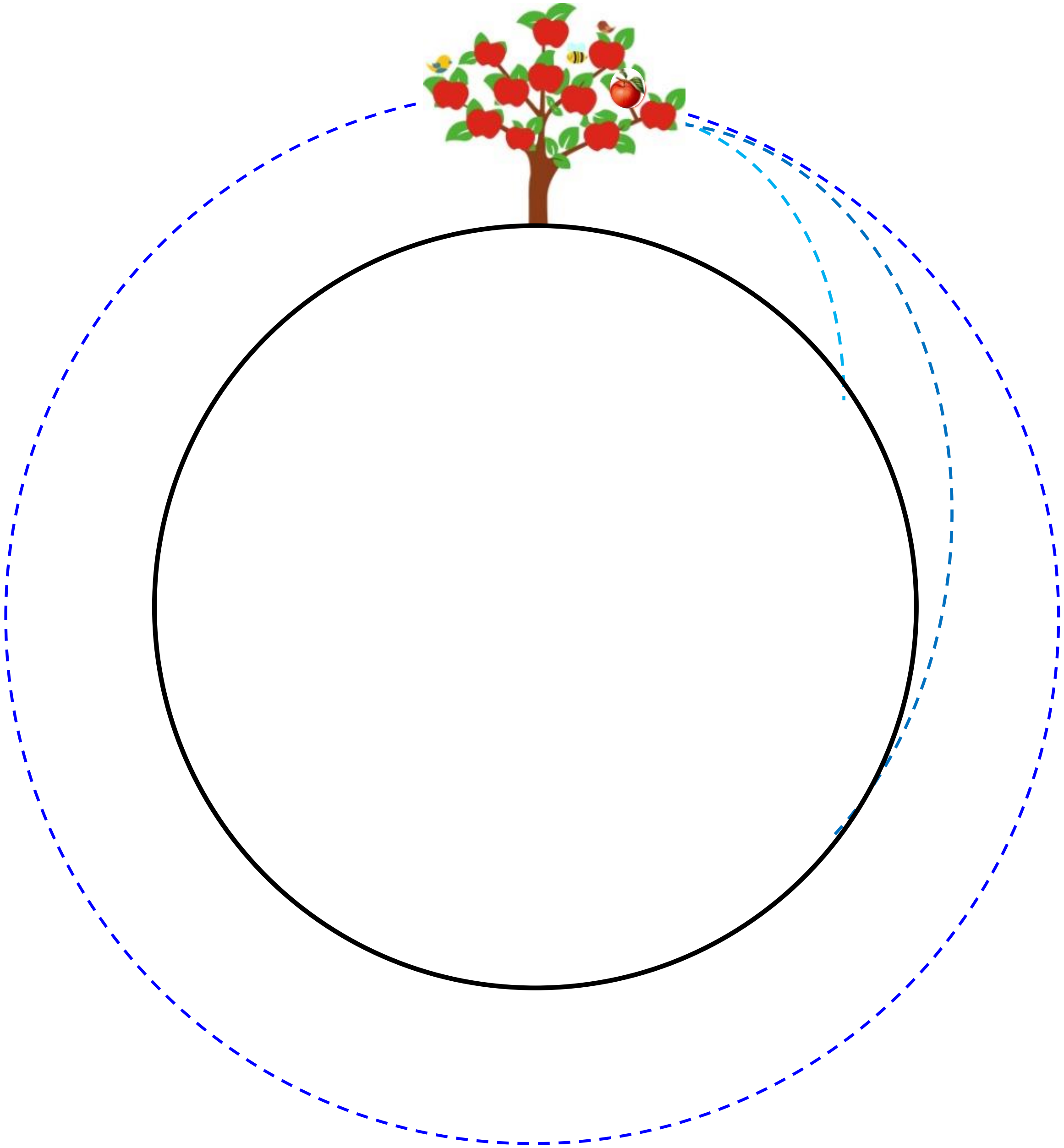
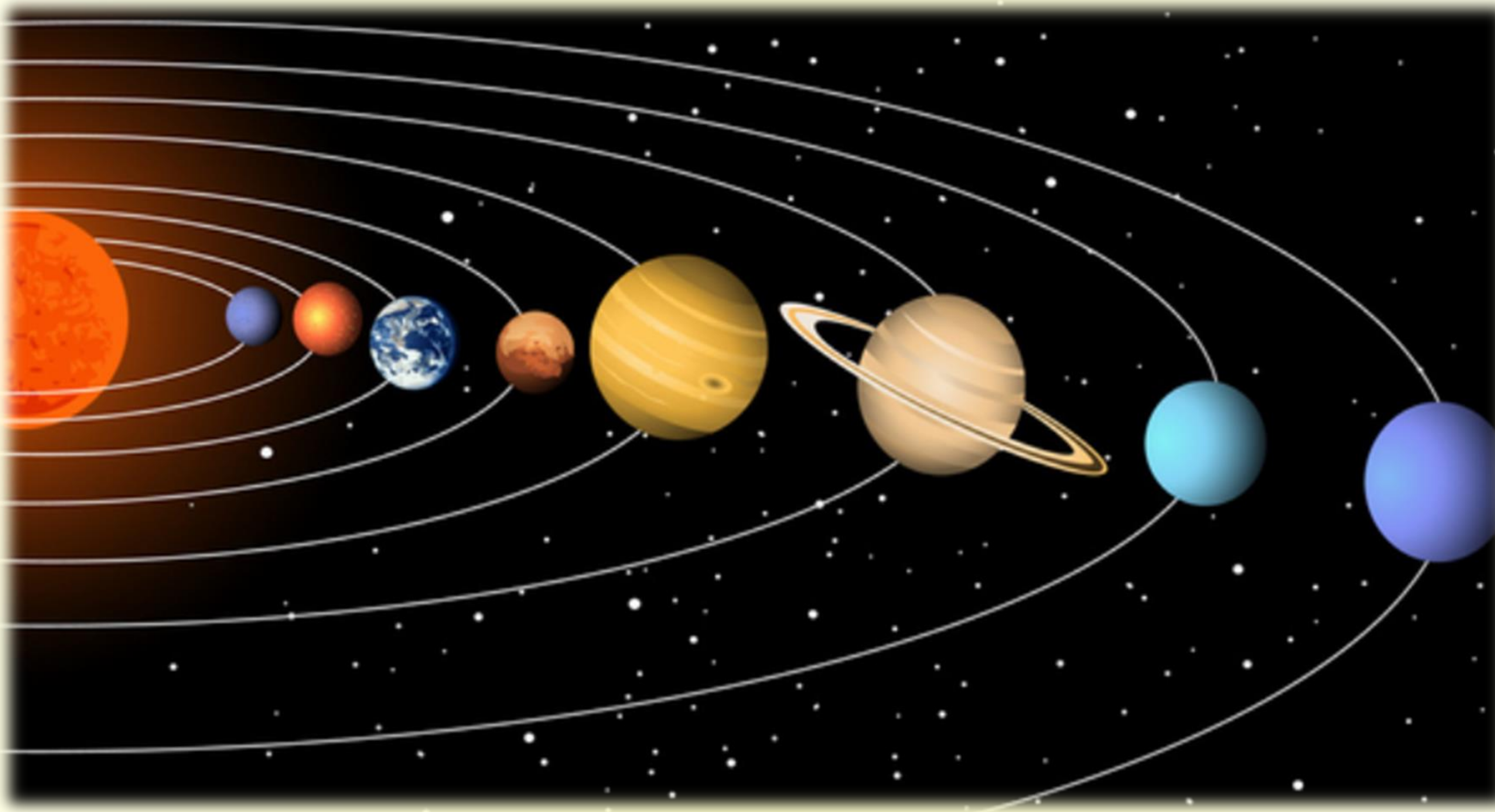
Chap4 Gravitation

Newton’s Thought Experiment







Galileo → Kepler → Descartes → Hooke → Newton

Newton's Law of Universal Gravitation

- **Statement:**

Every particle in the universe attracts every other particle with a force that is proportional to *the product of their masses* and *inversely* proportional to the *square of the distance* between them.
This force acts along the line joining the two particles.

- **Formula:** $$F_g = G\frac{m_1 m_2}{r^2}$$

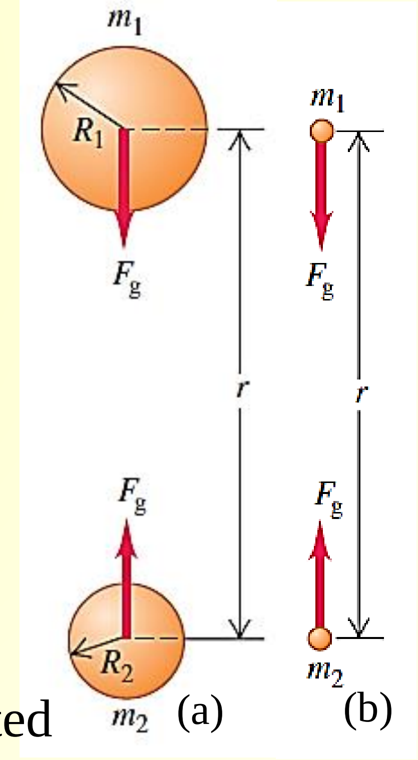
gravitational constant
($6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$)

  - universality
  - mutuality
  - independence
  - macroscopy

- **Gravitation of spherically symmetric bodies:**

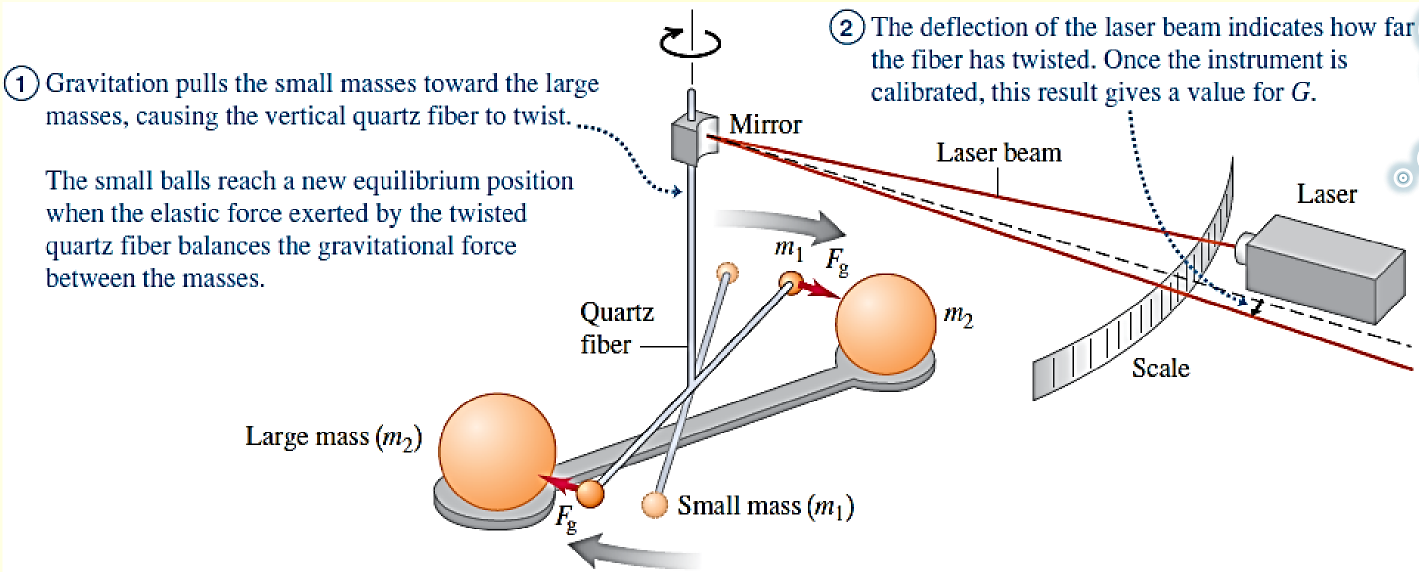
(a) F_g between two spherically symmetric masses m_1 & m_2…

(b) … is the same as if all the mass of each sphere is concentrated at the sphere's center.



➢ Determining the value of *G*:

Cavendish torsion balance



Test your Understanding of the Gravitation

The planet Saturn has about 100 times the mass of the Earth and is about 10 times farther from the Sun than the Earth is. Compared to the acceleration of the Earth caused by the Sun's gravitational pull, how great is the acceleration of Saturn due to the Sun's gravitation?

(A) 100 times greater **(B) 10 times greater** **(C) the same**

(D) $\frac{1}{10}$ as great **(E) $\frac{1}{100}$ as great** ✓

Weight

➢ **Extended definition:**

The *weight* of a body is *the total gravitational force* exerted on the body by all other bodies in the universe.

- **a body at the Earth's surface**

$$\left.\begin{aligned} w &= F_g \\ F_g &= G\frac{m_E m}{R_E^2} \\ w &= mg \end{aligned}\right\} \Rightarrow g = G\frac{m_E}{R_E^2}$$

(acceleration due to gravity at the Earth's surface)

$$\Downarrow$$

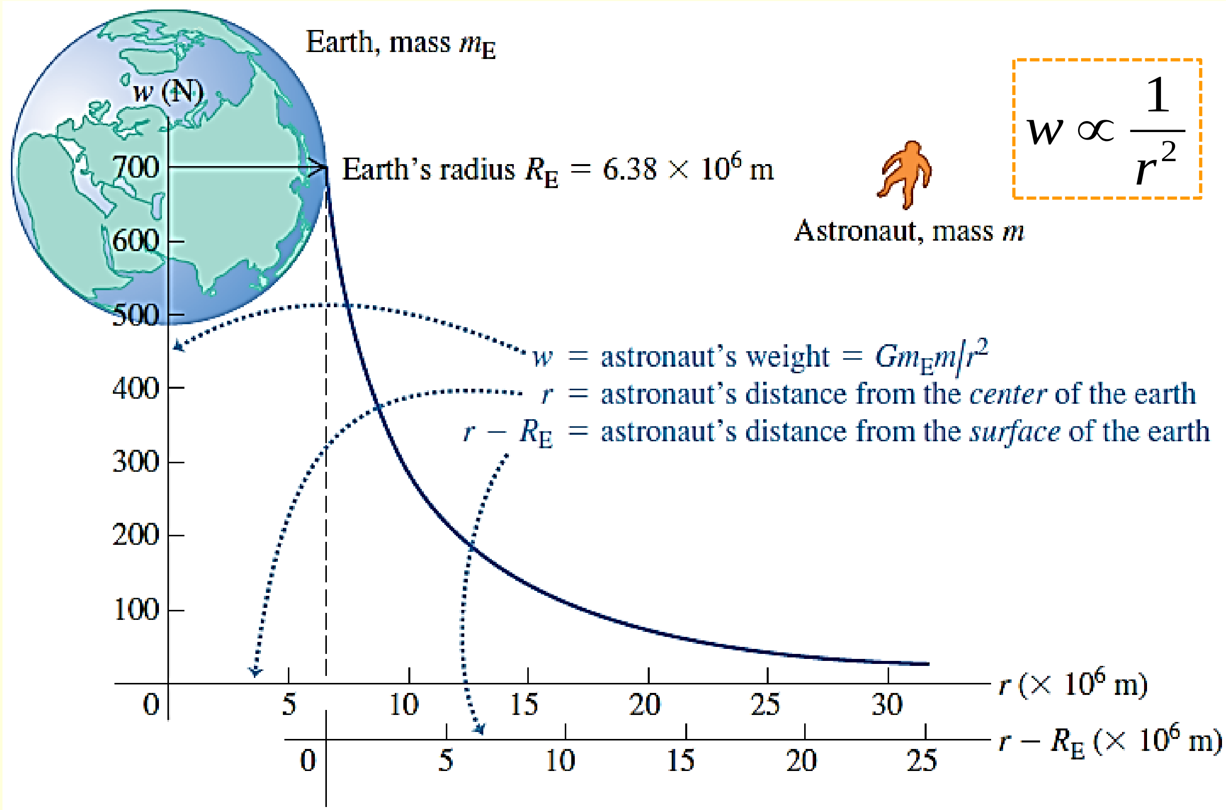
$$m_E = \frac{gR_E^2}{G} = \frac{9.8\times(6.38\times10^6)^2}{6.67\times10^{-11}}\,\text{kg} = 5.98\times10^{24}\,\text{kg}$$

- **a body at a point where the distance *r* from the center of the Earth ($r > R_E$)**

$$w = F_g = G\frac{m_E m}{r^2} \Rightarrow g = G\frac{m_E}{r^2}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$w \propto \frac{1}{r^2} \qquad\qquad g \propto \frac{1}{r^2}$$



Test your Understanding of the Acceleration due to Gravity

Rank the following hypothetical planets in order from highest to lowest value of *g* at the surface:

(i) mass = 2 times the mass of the earth, radius = 2 times the radius of the earth; ②

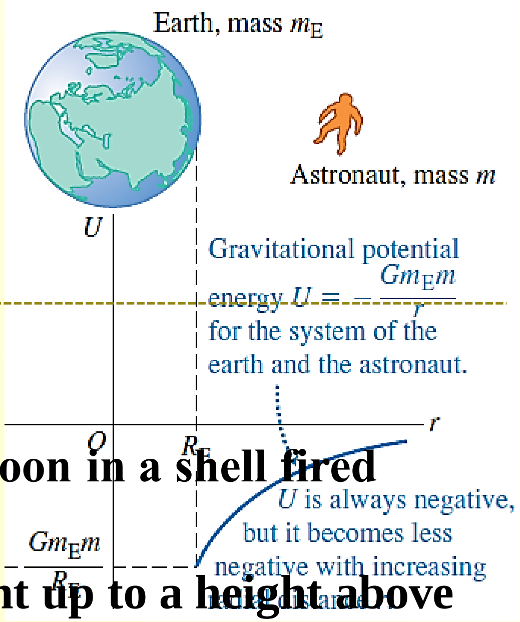
(ii) mass = 4 times the mass of the earth, radius = 4 times the radius of the earth; ③

(iii) mass = 4 times the mass of the earth, radius = 2 times the radius of the earth; ①

(iv) mass = 2 times the mass of the earth, radius = 4 times the radius of the earth. ④

Gravitational Potential Energy

$$U = -G\frac{m_E m}{r} \quad (r > R_E)\ (\text{set } r \to \infty,\ U = 0)$$



Example: "From the earth to the moon"

In Jules Verne's 1865 story with this title, three men went to the moon in a shell fired from a giant cannon sunk in the earth in Florida.

(a) Find the minimum muzzle speed needed to shoot a shell straight up to a height above the earth equal to the earth's radius R_E.

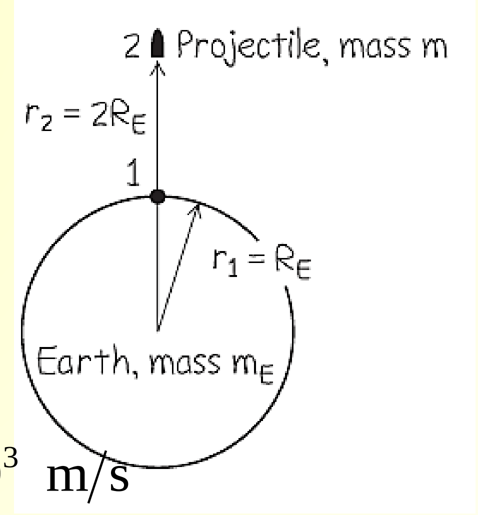
Neglect air resistance, the earth's rotation, and the gravitational pull of the moon. The earth's radius and mass are $R_E = 6.38\times10^6$ m and $m_E = 5.97\times10^{24}$ kg.

(a) Point 1 is at $r = R_E$, where the shell leaves the cannon with speed v_1 (*the target variable*); Point 2 is at $r_2 = 2R_E$, $v_2 = 0$, so $K_2 = 0$.

According to the energy-conservation equation,

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + \left(-\frac{Gm_Em}{R_E}\right) = 0 + \left(-\frac{Gm_Em}{2R_E}\right) \Rightarrow v_1 = \sqrt{\frac{Gm_E}{R_E}} = 7.9\times10^3 \text{ m/s}$$



Example: "From the earth to the moon"

In Jules Verne's 1865 story with this title, three men went to the moon in a shell fired from a giant cannon sunk in the earth in Florida.

(b) Find the minimum muzzle speed that would allow a shell to escape from the earth completely (the *escape speed*).

Neglect air resistance, the earth's rotation, and the gravitational pull of the moon. The earth's radius and mass are $R_E = 6.38 \times 10^6$ m and $m_E = 5.97 \times 10^{24}$ kg.

(b) In this case, $r_2 = \infty$, so $U_2 = 0$; $v_2 = 0$, so $K_2 = 0$

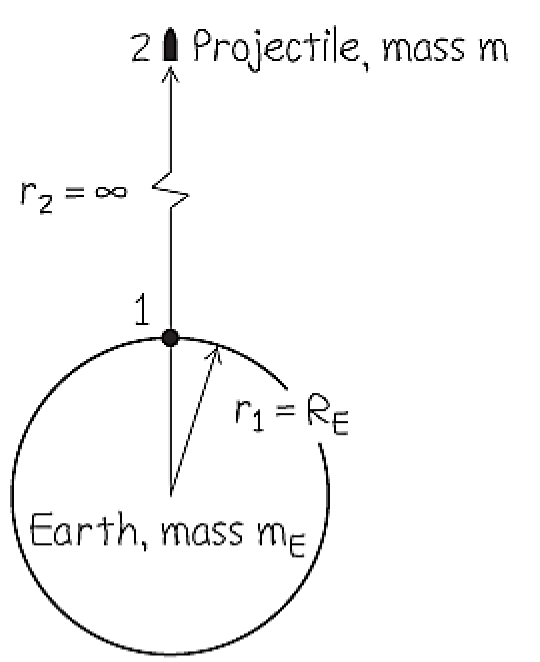
According to the energy-conservation equation,

$$K_1 + U_1 = K_2 + U_2$$

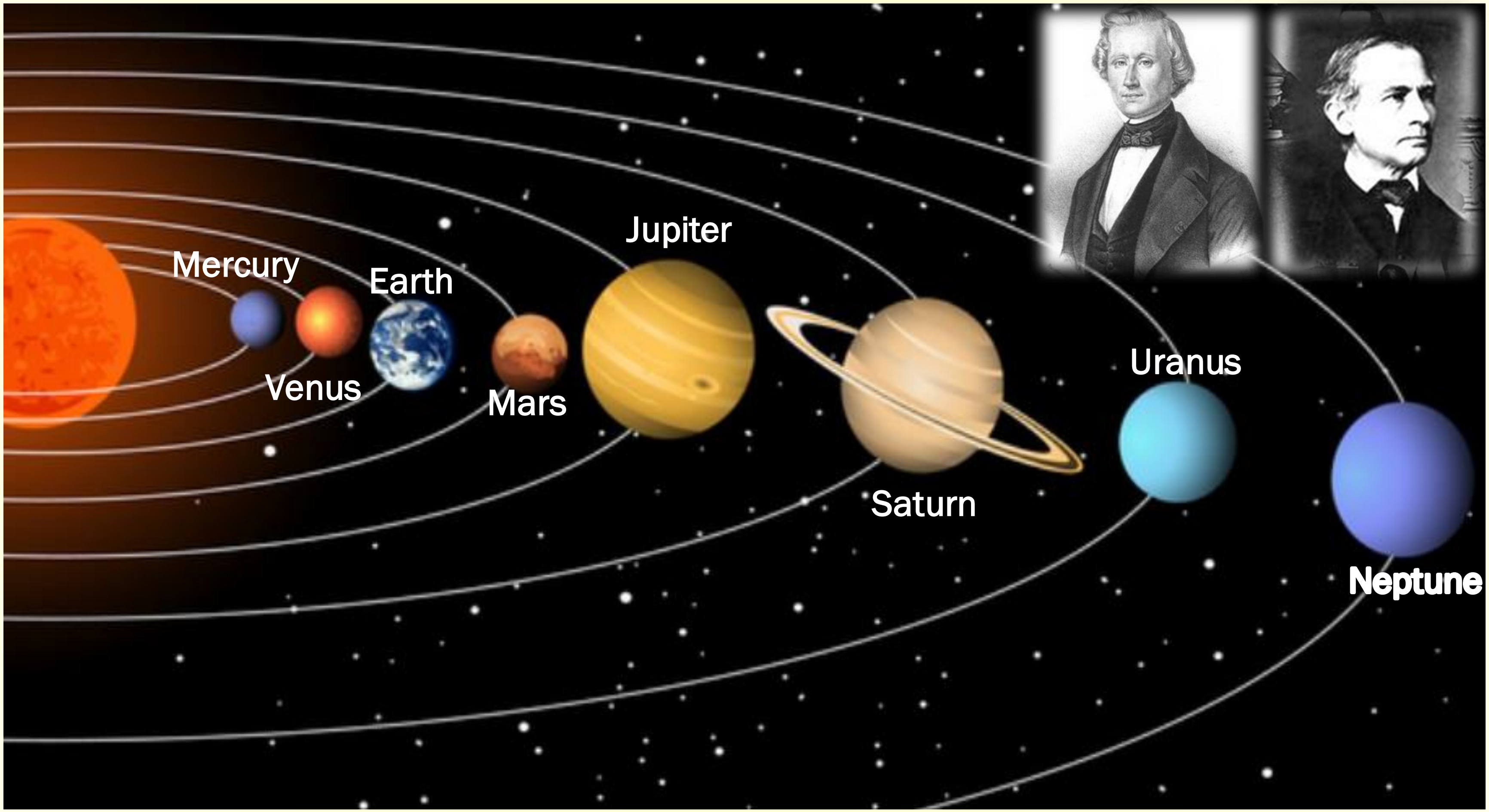
$$\frac{1}{2} m v_1^2 + \left(-\frac{G m_E m}{R_E}\right) = 0 + 0$$

escape velocity

$$v_1 = \sqrt{\frac{2 G m_E}{R_E}} = 1.12 \times 10^4 \text{ m/s}$$



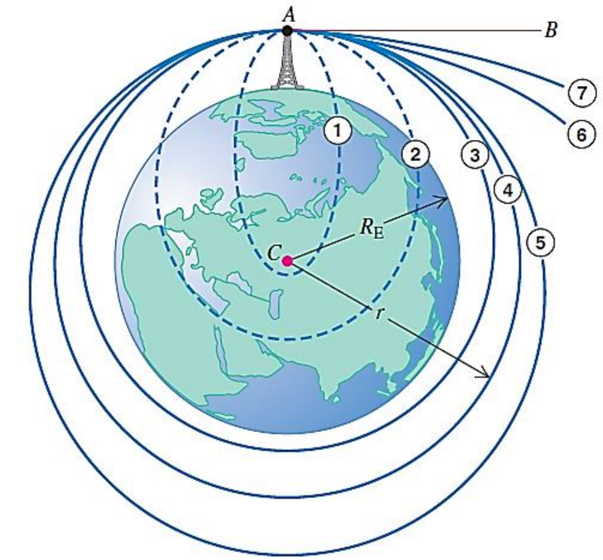
Application of Gravitation: Discovery of Unknown Planet



Application of Gravitation: Satellites

A projectile is launched from A toward B.

Trajectories ① through ⑦ show the effect of increasing initial speed.



➢ Satellites: Circular Orbits

$$\left.\begin{array}{l} F_c = m\dfrac{v^2}{r} \\ F_c = F_g \\ F_g = G\dfrac{m_E m}{r^2} \end{array}\right\} \Rightarrow v = \sqrt{\dfrac{Gm_E}{r}}$$

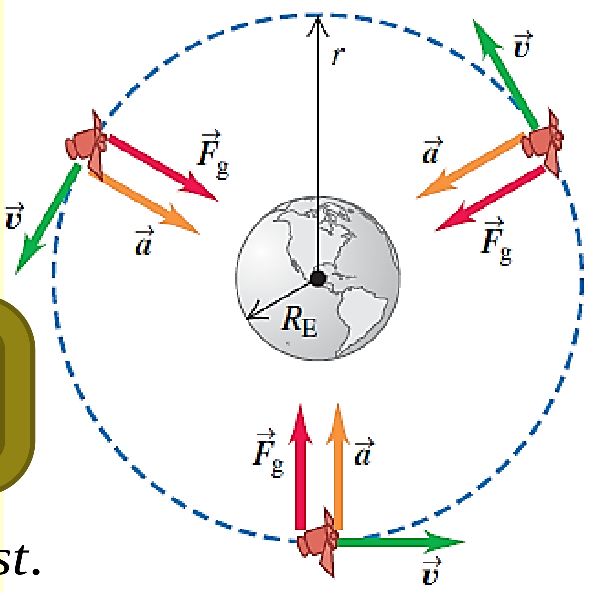
orbital velocity

$$\left.\begin{array}{l} v = \sqrt{\dfrac{Gm_E}{r}} \\ v = \dfrac{2\pi r}{T} \end{array}\right\} \Rightarrow T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

$$\frac{r^3}{T^2} = \frac{Gm_E}{4\pi^2} = const.$$

$$E = K + U = \frac{1}{2}mv^2 + \left(-\frac{Gm_E m}{r}\right) = -\frac{Gm_E m}{2r}$$

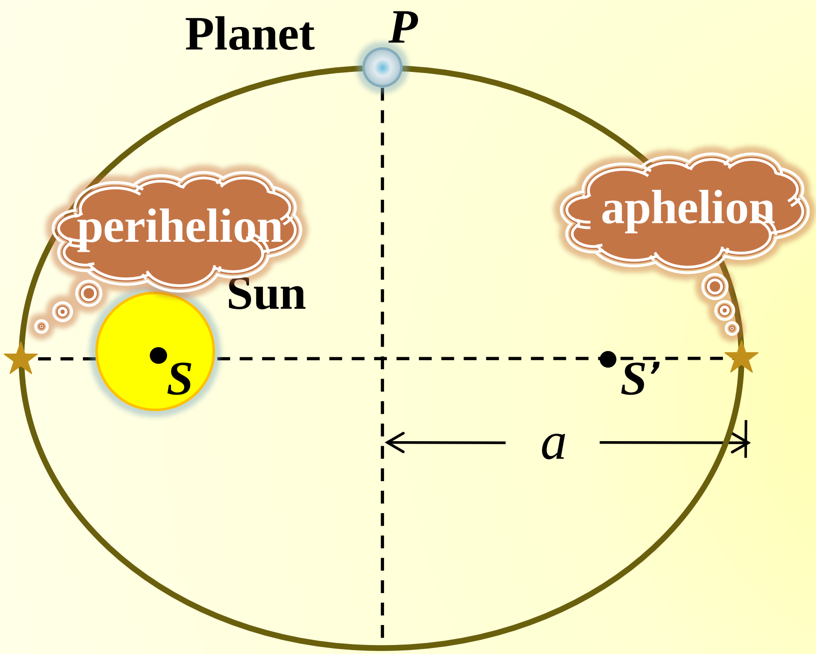
larger orbits
⇓
slower speeds & longer periods




Ptolemy
Copernicus
Tycho
Kepler
Galileo
Newton

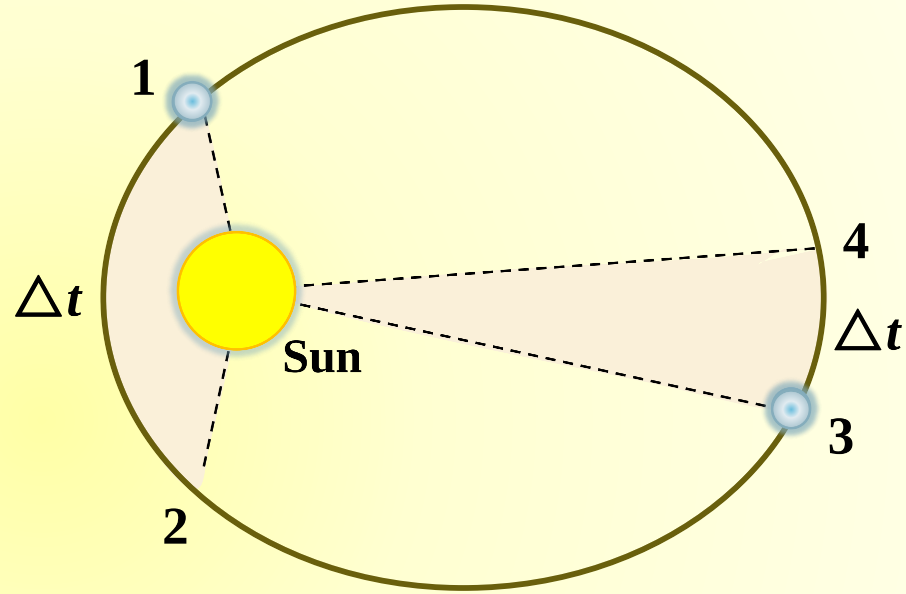
Kepler's Laws of Planetary Motion

✓ Kepler's 1st Law:



- elliptical orbit
- Sun at one focus

✓ Kepler's 2nd Law:



line *SP* sweeps out *equal* areas in *equal* times

✓ Kepler's 3rd Law:

$$\frac{a^3}{T^2} = const. \propto m_S$$