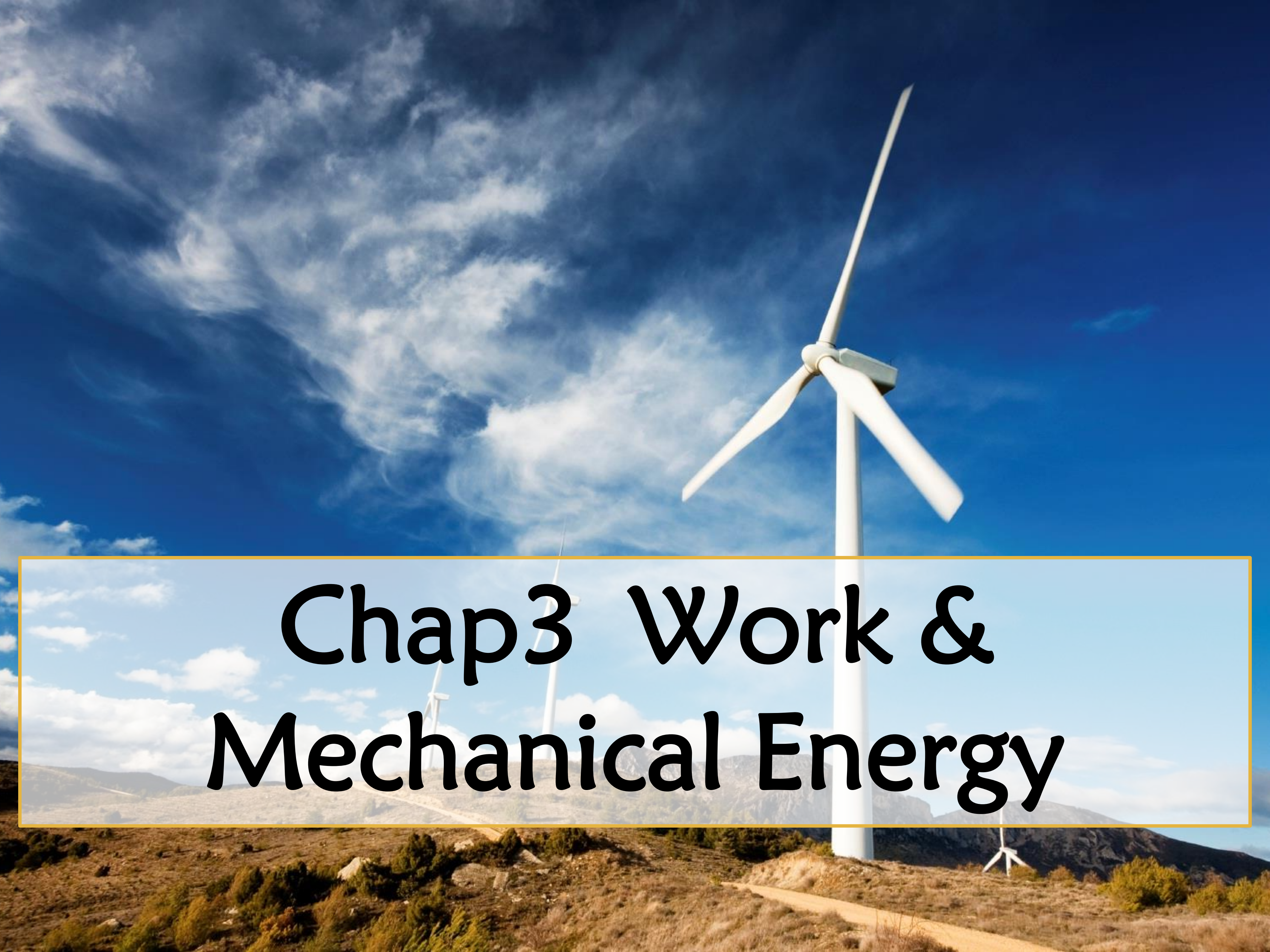
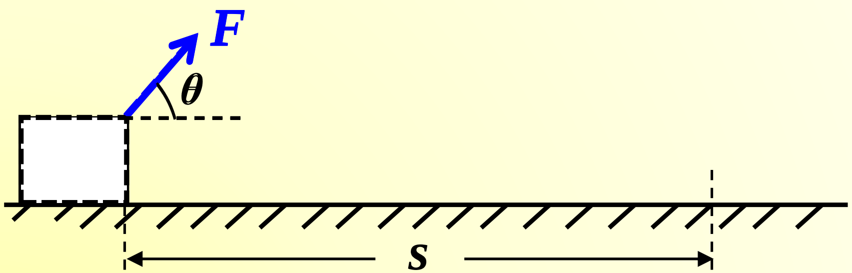
Chap3 Work & Mechanical Energy



Work ◦◦◦ *scalar*

➢ Work done by a constant force, straight-line motion:

- **Formula:** $W = Fs\cos\theta$
- **SI unit:** **J (1 J = 1 N·m)**



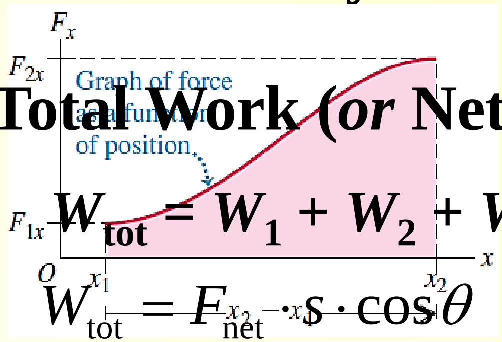
- **Work: Positive, Negative, *or* Zero**

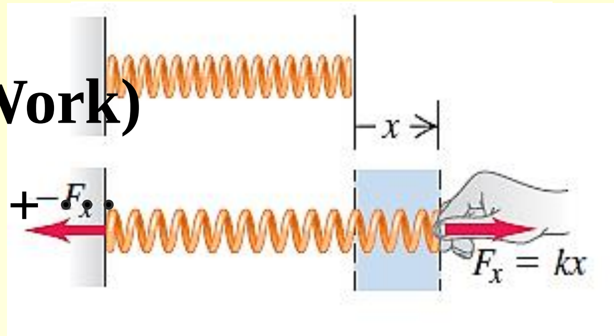
$0 \le \theta < 90^o,\ \cos\theta > 0,\ W > 0$ ➡ **positive work**

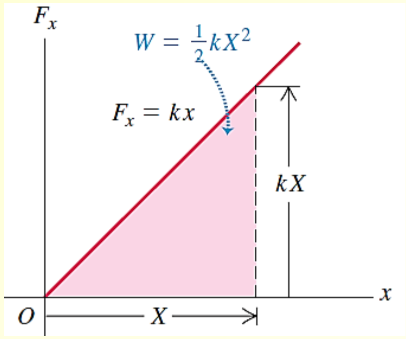
$90^o < \theta < 180^o,\ \cos\theta < 0,\ W < 0$ ➡ **negative work**

$\theta = 90^o,\ \cos\theta = 0,\ W = 0$ ➡ **NO work**

➢ Work done by a varying force, straight-line motion:







➢ Total Work (*or* Net Work)

- $W_{tot} = W_1 + W_2 + W_3 + \cdots$
- $W_{tot} = F_{net} \cdot s \cdot \cos\theta$

Power ◦◦◦ scalar

- **average power:** $P_{av} = \frac{\Delta W}{\Delta t} = F_{\parallel}\frac{\Delta s}{\Delta t} = F_{\parallel}v_{av} = Fv_{av}\cos\theta$ (F is constant)

  angle between F & S

- **instantaneous power:** $P = \lim_{\Delta t \to 0}\frac{\Delta W}{\Delta t} = \frac{dW}{dt} = Fv\cos\theta$ (F is constant or varying)

  angle between F & v

- **SI unit: W (1 W = 1 J/s)**

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1. An electron moves in a straight line toward the east with a constant speed of 8×10^7 m/s. It has electric, magnetic, and gravitational forces acting on it. During a 1-m displacement, the total work done on the electron is

(A) positive **(B) negative** **(C) zero** ✓

(D) not enough information given to decide

2. A force of 150 N is required to keep an object sliding at a constant speed of 2 m/s across a rough floor. How much power is being expended to maintain this motion?

(A) 50 W **(B) 300 W** ✓ **(C) 200 W** **(D) 400 W**

Kinetic Energy

scalar

- **Formula:** $K = \frac{1}{2}mv^2$
- **SI unit:** **J (1 J = 1 kg·m²/s² = 1 N·m)**

Work-Energy Theorem

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$ (*valid in general*)

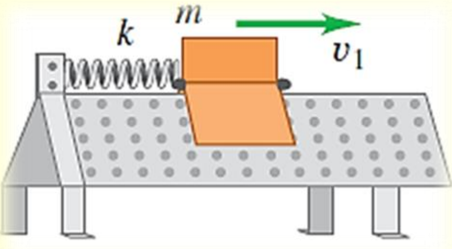
An block of mass *m* is traveling at constant speed *v* in a circular path of radius *r*. How much work is done by the centripetal force during one half of a revolution?

(A) $\pi m v^2$ **(B)** $2\pi m v^2$ **(C)** 0 ✓

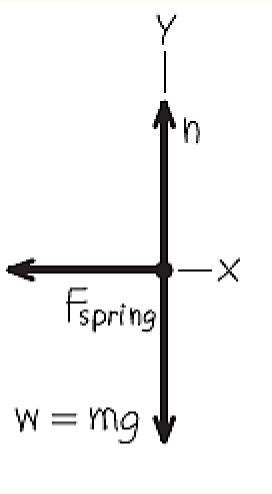
(D) $3\pi m v^2 r$ **(E)** $2\pi m v^2 r$

Example: Motion with a varying force

An air-track glider of mass 0.1 kg is attached to the end of a horizontal air track by a spring with force constant 20.0 N/m. Initially the spring is unstretched and the glider is moving at 1.5 m/s to the right. Find the maximum distance *d* that the glider moves to the right if the air track is turned on, so that there is no friction.



As the glider moves from $x_1 = 0$ to $x_2 = d$, the total work done on the glider is: $W_{tot} = W_{F_{spring}} = -\frac{1}{2}kd^2$



The initial kinetic energy is: $K_1 = \frac{1}{2}mv_1^2$

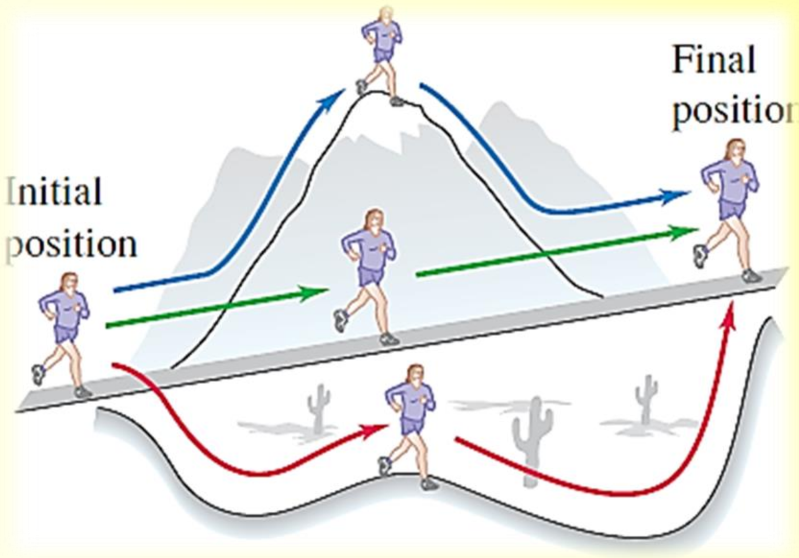
The final kinetic energy is: $K_2 = 0$

According to the work-energy theorem,

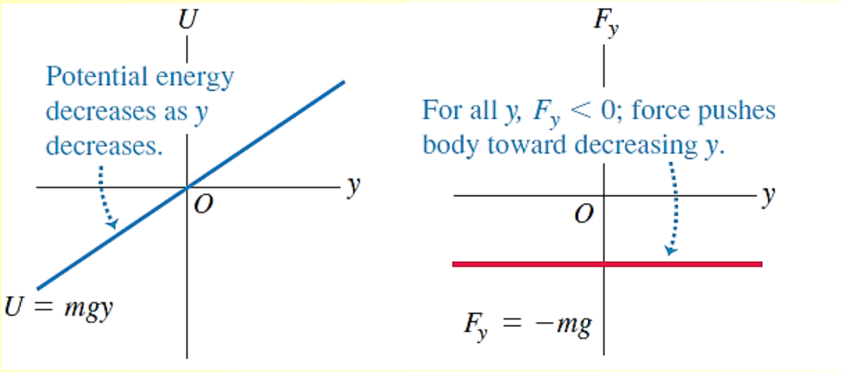
$$W_{tot} = K_2 - K_1 \Rightarrow -\frac{1}{2}kd^2 = -\frac{1}{2}mv_1^2 \Rightarrow d = \sqrt{\frac{m}{k}}v_1 = 10.6 \text{ cm}$$

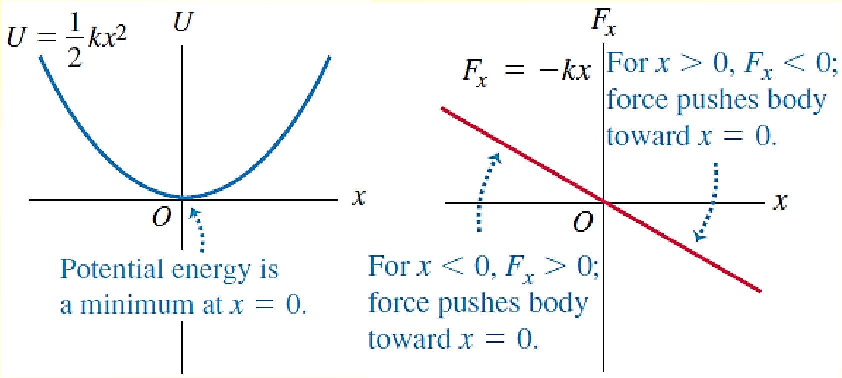
Potential Energy scalar

- **Gravitational Potential Energy:** $U_{grav} = mgy$ $W_{grav} = -\Delta U_{grav}$
- **Elastic Potential Energy:** $U_{el} = \frac{1}{2}kx^2$ $W_{el} = -\Delta U_{el}$
- **Conservative Force:**



Because the gravitational force is conservative, the work it does is the same for all three paths.





Mechanical Energy

- ✓ **Conservation of Mechanical Energy:**

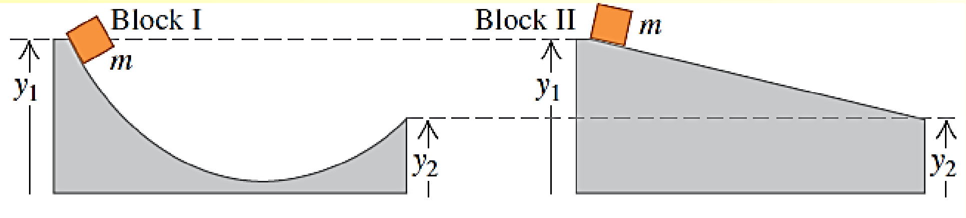
$$E = K + U = const.$$ (*Only conservative forces do work*)

- ✓ **If nonconservative forces are present,**

$$\left.\begin{aligned} W_{nc} + W_{con} &= K_2 - K_1 \\ W_{con} &= -\Delta U = U_1 - U_2 \end{aligned}\right\} \Rightarrow K_1 + U_1 + W_{nc} = K_2 + U_2$$

Test Your Understanding of this Section

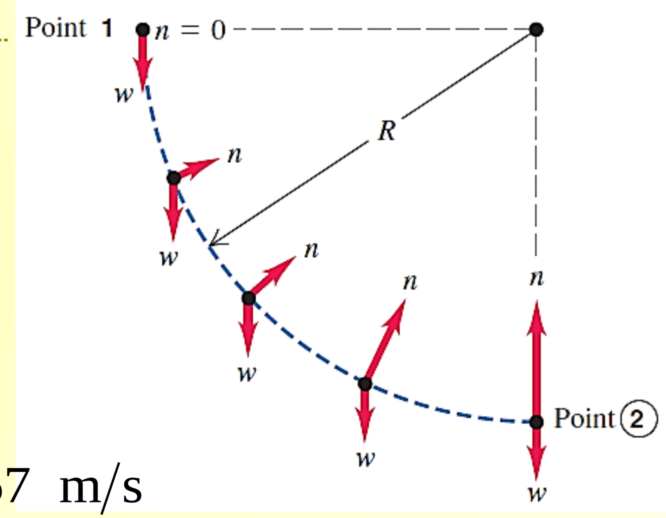
The figure shows two different frictionless ramps. The height y_1 and y_2 are the same for both ramps. If a block of mass *m* is released from rest at the left-hand end of each ramp, which block arrives at the right-hand end with the greater speed?



(i) block I (ii) block II (iii) the speed is the same for both blocks

Example: Speed at the bottom of a vertical circle

Your cousin Throckmorton skateboards from rest down a curved, frictionless ramp. If we treat Throcky and his skateboard as a particle, he moves through a quarter-circle with radius *R* = 3 m. Throcky and his skateboard have a total mass of 25 kg. (a) Find his speed at the bottom of the ramp. (b) Find the normal force that acts on him at the bottom of the curve.



(a) The various energy quantities are

$$K_1 = 0,\ U_{grav,1} = mgR$$

$$K_2 = \frac{1}{2}mv_2^2,\ U_{grav,2} = 0$$

From conservation of mechanical energy,

$$K_1 + U_{grav,1} = K_2 + U_{grav,2} \Rightarrow v_2 = \sqrt{2gR} = 7.67\ \text{m/s}$$

(b) At point 2, the centripetal acceleration is toward the center of the circle and has magnitude: $a_c = \frac{v_2^2}{R} = 2g$

According to Newton's second law, $\Sigma F_y = n - w = ma_c \Rightarrow n = 3mg = 735\text{N}$

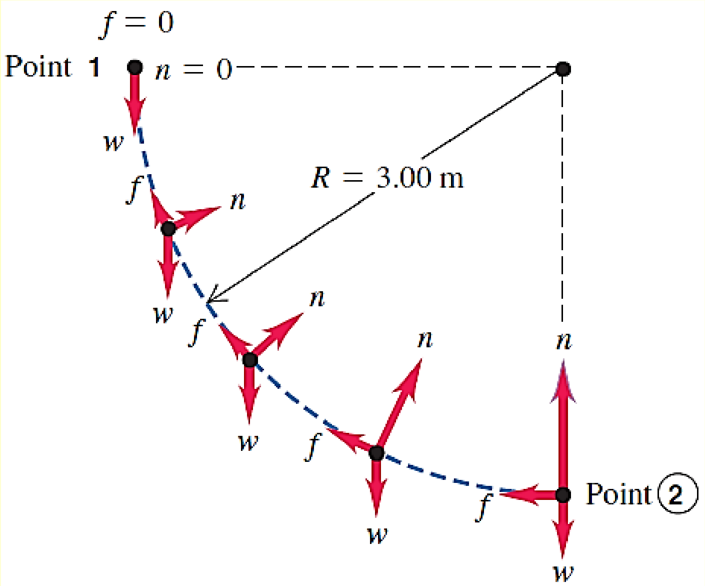
Example: A vertical circle with friction

Suppose that the ramp of previous Example is not frictionless, and that Throcky's speed at the bottom is only 6.00 m/s, not the 7.67 m/s we found previously. What work was done on him by the friction force?

The various energy quantities are

$$K_1 = 0,\ U_{grav,1} = mgR = 735\text{J}$$

$$K_2 = \frac{1}{2}mv_2^2 = 450\text{J},\ U_{grav,2} = 0$$



Friction force (nonconservative force) does work on Throcky from point 1 to point 2, and thus

$$K_1 + U_{grav,1} + W_f = K_2 + U_{grav,2}$$

$$W_f = K_2 + U_{grav,2} - K_1 - U_{grav,1} = -285\ \text{J}$$

The friction force does negative work on Throcky as he descends, so the total mechanical energy decreases.