1. Which one of the following choices correctly represents a length of $3.00\ mm$?

(A) $3.00 \times 10^{-6}\ m$
(B) $3.00 \times 10^{-3}\ m$
(C) $3.00 \times 10^{-2}\ m$
(D) $3.00 \times 10^{3}\ m$
(E) $3.00 \times 10^{6}\ m$

B… milli represents 10^{-3}

2. A box uniformly slides $7.50\ m$ to rest across a flat surface in a time of $12.0\ s$. What was the initial speed of the box when it started its slide?

(A) $0.313\frac{m}{s}$ **(B)** $0.625\frac{m}{s}$ **(C)** $1.25\frac{m}{s}$ **(D)** $2.50\frac{m}{s}$ **(E)** $5.00\frac{m}{s}$

C… METHOD #1: Using constant acceleration kinematics, we have the average velocity computed as $\langle v\rangle = \frac{\Delta x}{\Delta t} = \frac{7.5}{12} = 0.625\frac{m}{s}$ which leads to $\langle v\rangle = \frac{1}{2}(v_0 + v_f) \rightarrow 0.625 = \frac{1}{2}(v_0 + 0) \rightarrow v_0 = 1.25\frac{m}{s}$.
METHOD #2: One can graph velocity vs time for the box. Knowing that the area under the curve is the change in position, we have $A = \frac{1}{2}bh \Rightarrow 7.5 = \frac{1}{2}(12.0)v_0 \Rightarrow v_0 = 1.25\frac{m}{s}$

3. Which one of the following quantities is not a vector quantity?

(A) Average speed **(B)** Average velocity **(C)** Linear momentum **(D)** Acceleration **(E)** Average force

A… Vector quantities point… and speed is a scalar.

4. A standing wave on a string is produced. Which one of the following choices best describes the location on the string at which maximum constructive interference occurs?

(A) node **(B)** antinode **(C)** harmonic **(D)** overtone **(E)** amplitude

B… Locations of complete constructive interference are known as antinodes on the standing wave.

5. An object initially is moving upward while in free fall. Which one of the following choices best represents the direction of the object's acceleration during its flight?

	Object moving upward	*Object at peak of motion*	*Object moving downward*
(A)	Upward	Zero	Downward
(B)	Downward	Zero	Downward
(C)	Upward	Downward	Downward
(D)	Downward	Zero	Upward
(E)	Downward	Downward	Downward

E… The gravitational force is always directed downward!!

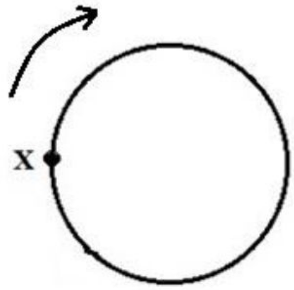
6. A car travels at $20.0\frac{miles}{hr}$. Which one of the following choices best represents the speed of the car in SI units of $\frac{m}{s}$?

(A) $533\frac{m}{s}$ **(B)** $45.0\frac{m}{s}$ **(C)** $20.0\frac{m}{s}$ **(D)** $8.9\frac{m}{s}$ **(E)** $0.75\frac{m}{s}$

D… Converting units… $20\frac{miles}{hr} \times \frac{1\ hr}{60\ \text{min}} \times \frac{1\text{min}}{60\ s} \times \frac{1600\ m}{1\ mi} = 8.9\frac{m}{s}$

7. An object moves clockwise with constant speed around the vertical circle shown. Which arrow best indicates the direction of the object's instantaneous acceleration at the point labeled X?

	Acceleration at X
(A)	→
(B)	↓
(C)	←
(D)	↘
(E)	There is no acceleration.



A… Objects that move in uniform circular have a velocity direction change toward the center of the circle being traced out.

8. A negatively charged balloon remains at rest when placed on a vertical wall. Which one of the following terms is most closely associated with the electrical phenomenon allowing the balloon to remain on the wall?

(A) Radiation **(B)** Grounding **(C)** Reduction **(D)** Current **(E)** Polarization

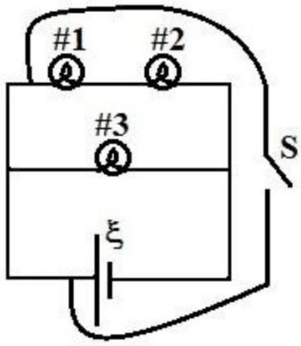
E… After the balloon obtains charge from being rubbed, it causes polarization in the wall allowing like charges to be closer to each other when they come in contact, providing an attractive force to prevent the balloon from falling to the ground.

9. Two cars are moving to the right on a horizontal track, each with constant acceleration. At an instant of time, the information about the cars is shown:
Car #1: position = 125.0 m; velocity = $13.0\frac{m}{s}$; constant acceleration = $1.5\frac{m}{s^2}$
Car #2: position = 80.0 m; velocity = $9.30\frac{m}{s}$; constant acceleration = $5.5\frac{m}{s^2}$
During the next 1.0 s of motion, which one of the following choices best represents what happens to the distance between the cars?

(A) It decreases during the entire 1.0 second of motion.
(B) It increases during the entire 1.0 second of motion.
(C) It initially increases and then decreases resulting in a greater distance between the cars after 1.0 second.
(D) It initially increases and then decreases resulting in a smaller distance between the cars after 1.0 second.
(E) It initially increases and then decreases resulting in the same distance between the cars after 1.0 second.

C… Since $v_{car_1} > v_{car_2}$ at the start of the interval, the distance between the cars increases. However, noting at that the end of the second that the speed of the cars are $v_{car_1} = v_0 + at = 13 + (1.5)(1) = 14.5\frac{m}{s}$ and $v_{car_2} = 9.3 + (5.5)(1) = 14.8\frac{m}{s}$, by the interval's end, car 2 is moving faster than car 1 and catching up to it. Hence, the distance between cars is now decreasing. Since the speed of car 2 is only slightly greater than car 1 at the end and started off much slower, then car 1 will actually increase its distance from car 2. To check, one can compute the position changes as $\Delta x_1 = v_1 t + \frac{1}{2}a_1 t_1^2 = (13)(1) + \frac{1}{2}(1.5)(1)^2 = 13.75\ m$ with $\Delta x_2 = v_2 t + \frac{1}{2}a_2 t_2^2 = (9.3)(1) + \frac{1}{2}(5.5)(1)^2 = 12.05\ m$.

10. For the circuit shown, the three light bulbs have identical resistance R, the battery is ideal, and all wires have no resistance. Which one of the following choices correctly identifies the light bulbs that either become dimmer or go out completely when the switch, S, in the circuit is closed?



(A) All 3 bulbs
(B) Bulbs #1 and #2 only
(C) Bulb #3 only
(D) Bulb #1 only
(E) None of the bulbs

E… When the switch is closed, a wire of zero resistance is added to the circuit. We note, though, that the wire is connected to the left side of the battery. In other words, we have effectively added a shorting wire in parallel to the other resistance-less wires on the left side of the circuit. There is no effect on any of the bulbs as a result because the circuit is effectively unchanged.

11. Two length measurements are made and recorded as $L_1 = 84.55\ cm$ and $L_2 = 33.55\ cm$. Two other length measurements are made and recorded as $L_3 = 1.750\ cm$ and $L_4 = 1.250\ cm$. These measurements are used to compute the quantity $(L_1 + L_2) - (L_3 + L_4)$ using the rules of significant figures. Which one of the following choices best represents the correct result to this calculation?

(A) 115.100 cm **(B)** 115.10 cm **(C)** 115.1 cm **(D)** 115 cm **(E)** 120 cm

B… When adding/subtracting, the key is to line up the values and look at the last column of digits for which each quantity has a significant digit. That is, $L_1 + L_2 = 118.10\ cm$ and $L_3 + L_4 = 3.000\ cm$. Upon subtracting, we have then $118.10 - 3.000 = 115.10\ cm$ since we cannot keep the thousandths place in the result since 118.10 only has values to the hundredths place!

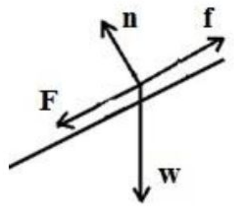
12. A box of mass 12.0 kg is being pushed to the right across a horizontal surface. When the box has 12.0 J of kinetic energy, a 12.0 N net force acts on it. Which one of the following choices best represents the magnitude of the linear momentum of the box at this instant?

(A) 6.00 $kg\frac{m}{s}$ **(B)** 8.50 $kg\frac{m}{s}$ **(C)** 12.0 $kg\frac{m}{s}$ **(D)** 17.0 $kg\frac{m}{s}$ **(E)** 24.0 $kg\frac{m}{s}$

D… METHOD #1: Knowing that the linear momentum is computed as $p = mv$, we need to find the speed. This is obtained from the kinetic energy as $KE = \frac{1}{2}mv^2 \rightarrow 12 = \frac{1}{2}(12)v^2 \rightarrow v = \sqrt{2}\frac{m}{s}$. So, $p = mv = (12)(\sqrt{2}) = 17.0\ kg\frac{m}{s}$.

METHOD #2: By rewriting the kinetic energy slightly, one has the form $KE = \frac{p^2}{2m}$ and so $p = \sqrt{2m\ KE} = \sqrt{2(12)(12)} = 17.0\ kg\frac{m}{s}$

13. An object is being pushed at constant speed on an inclined plane. The free body diagram of the object is shown with the gravitational force represented by W, the friction force by f, the applied external push parallel to the incline by F, and the normal force with the surface by n. Which one of the following choices represents correct relationships between the forces?



(A) $n > W$ and $F < f$
(B) $n < W$ and $F = f$
(C) $n < W$ and $F < f$
(D) $n = W$ and $F > f$
(E) $n = W$ and $F = f$

C… If we rotate our coordinates slightly to take advantage of the incline, we find in the direction perpendicular to the incline that the total normal force acts off the incline and only a portion of the gravitational force acts that way. That is, $n < w$. Also, along the incline's surface, there are two downward forces (applied and a piece of the gravitational) and one upward force. Since there is no acceleration, these forces sum to zero N. Hence, $F + w_{along\ incline} = f$ meaning that $F < f$.

14. A particle has a position, x, as a function of time, t, given as $x(t) = -15 - 25t + 10t^2$. Which one of the following choices represents the magnitude of the particle's acceleration? All quantities are expressed in base SI units.

(A) $5\frac{m}{s^2}$ **(B)** $10\frac{m}{s^2}$ **(C)** $15\frac{m}{s^2}$ **(D)** $20\frac{m}{s^2}$ **(E)** $40\frac{m}{s^2}$

D… When the position takes the form given, we have constant acceleration with $x = x_0 + v_0t + \frac{1}{2}at^2$. By identifying terms, we see that $\frac{1}{2}a = 10$ or that $a = 20\frac{m}{s^2}$.

15. A skydiver falls downward through the air with constant speed. Which one of the following choices correctly describes the Newton's Third Law pair force to the air resistance acting on the skydiver during the fall?

(A) There is no Third Law pair force for this kind of situation.
(B) The gravitational force acting on the skydiver by the Earth.
(C) The force that molecules in the air exert on neighboring molecules in the air.
(D) The force exerted on molecules in the air by the ground.
(E) The force exerted on molecules in the air by the skydiver.

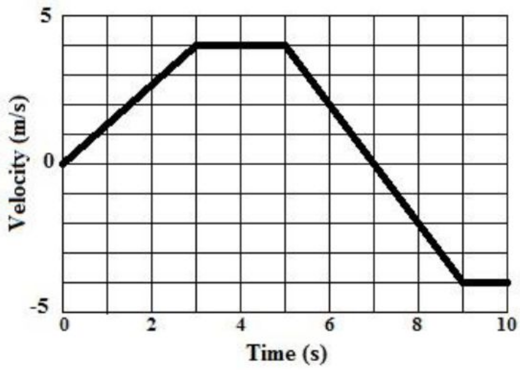
E… The molecules in the air exert a force upward on the skydiver….

16. "Particles of matter also have associated wavelengths and can behave as waves." To which scientist is this concept attributed?

(A) de Broglie **(B)** Pauli **(C)** Fermi **(D)** Heisenberg **(E)** Rydberg

A… This is the de Broglie hypothesis.

17. An object starts at the origin and its velocity along a line vs. time is graphed. Which one of the following choices best gives the proper interval(s) of time for which the object is moving away from the origin?



(A) Only for times $0\ s < t < 3\ s$
(B) Only for times $0\ s < t < 5\ s$
(C) Only for times $3\ s < t < 5\ s$
(D) Only for times $0\ s < t < 7\ s$
(E) For times $0\ s < t < 3\ s$ and $5\ s < t < 9\ s$

D… With the positive velocity, the object is moving "to the right" arbitrarily. The total change in position is from the area under the curve. Hence, as long as the velocity is positive, the object is moving away from the origin. The negative slope from 5 to 7 seconds simply indicates that the object moves forward but is slowing down (like a vehicle approaching a stop sign).

18. A 680 Hz tuning fork is placed over a tube open at both ends that is filled with air. As a result, a standing wave in the 3rd harmonic is produced. The speed of sound in air is $340\frac{m}{s}$. What is the length of the tube?

(A) $0.38\ m$ **(B)** $0.67\ m$ **(C)** $0.75\ m$ **(D)** $1.33\ m$ **(E)** $1.50\ m$

C... For an open tube, we can use that $f_n = \frac{nv}{2L} \rightarrow L = \frac{nv}{2f_n} \rightarrow L = \frac{(3)(340)}{2(680)} = 0.75\ m$

19. Ten moles of helium gas are enclosed in a container at a pressure of $1.00\ atm$ and at a temperature of $400\ K$. Which one of the following choices best represents the density of this gas sample?

(A) $0.012\frac{kg}{m^3}$ **(B)** $0.12\ \frac{kg}{m^3}$ **(C)** $1.2\frac{kg}{m^3}$ **(D)** $120\frac{kg}{m^3}$ **(E)** $1.2 \times 10^4\frac{kg}{m^3}$

B… From $PV = nRT$, rearrange to compute the volume of the gas as $V = \frac{nRT}{P} = 0.328\ m^3$. The total mass is $m = nM = (10)\left(0.004\frac{kg}{mol}\right) = 0.040\ kg$. Finally, $\rho = \frac{m}{V} = \frac{0.040}{0.328} = 0.12\frac{kg}{m^3}$.

20. Which one of the following choices best represents the work for which the 2014 Nobel Prize in Physics was awarded?

(A) Landing the Rosetta Philae Lander on the surface of Comet 67P/Churyumov-Gerasimenko
(B) The detection of dark matter
(C) The invention of the blue LED
(D) The creation of a tractor beam using sound
(E) The detection of neutrinos from the Sun which agreed with theory

C… The Nobel Prize was awarded to Shuji Nakamura, Hiroshi Amano, and Isamu Akasaki for the invention of the blue LED.

21. An object is launched from the ground at an angle of 60° above the horizontal with a speed of $20.0\frac{m}{s}$. What is the magnitude of the average velocity of the object from just after launch until it reaches its highest vertical position during flight?

(A) $13.7\frac{m}{s}$ **(B)** $13.2\frac{m}{s}$ **(C)** $10.0\frac{m}{s}$ **(D)** $9.3\frac{m}{s}$ **(E)** $8.7\frac{m}{s}$

B… METHOD #1: By breaking the initial velocity into components, we have $v_x = 20\cos 60^\circ = 10\frac{m}{s}$ and $v_y = 20\sin 60^\circ = 17.3\frac{m}{s}$. And so, since the acceleration is only in the y-direction, $\langle v_x\rangle = 10\frac{m}{s}$ and $\langle v_y\rangle = \frac{1}{2}(v_{0y} + v_y) = \frac{1}{2}(17.3 + 0) = 8.66\frac{m}{s}$. So, the total average velocity is computed using the Pythagorean Theorem giving $|\langle v\rangle| = \sqrt{\langle v_x\rangle^2 + \langle v_y\rangle^2} = \sqrt{10^2 + 8.66^2} = 13.2\frac{m}{s}$.

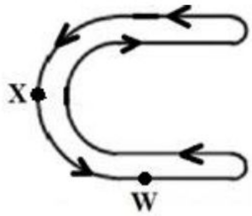
METHOD #2: Again, we start by breaking the velocity into components, but go about finding the location of the highest point in the trajectory. From the vertical component of motion, we have $v_y = v_{0y} + at \rightarrow 0 = 17.3 + (-10)t \rightarrow t = 1.73\ s$. Now, $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \rightarrow y = 0 + (17.3)(1.73) + \frac{1}{2}(-10)(1.73)^2 = 15.0\ m$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \rightarrow x = 0 + (10)(1.73) + 0 = 17.3\ m$. Hence, the magnitude of the displacement of the object is $\sqrt{17.3^2 + 15.0^2} = 22.9\ m$. And finally from the definition of average velocity, we compute $|\langle \vec{v}\rangle| = \left|\frac{\Delta\vec{r}}{\Delta t}\right| = \frac{22.9\ m}{1.73\ s} = 13.2\frac{m}{s}$

22. Satellite 1 makes a circular orbit around the Earth with a radius $r_1 = R$.
Satellite 2 makes a circular orbit around the Earth with a radius $r_2 = 2R$.
We let v represent the speed of a satellite and a represent the magnitude of a satellite's acceleration. Which one of the following choices gives the correct relation between the speeds and accelerations of the satellites?

(A) $v_2 = \frac{1}{\sqrt{2}}v_1$; $a_2 = \frac{1}{4}a_1$
(B) $v_2 = \frac{1}{2}v_1$; $a_2 = \frac{1}{4}a_1$
(C) $v_2 = \frac{1}{\sqrt{2}}v_1$; $a_2 = \frac{1}{2}a_1$
(D) $v_2 = \frac{1}{2}v_1$; $a_2 = \frac{1}{2}a_1$
(E) $v_2 = v_1$; $a_2 = \frac{1}{2}a_1$

A… Making the free body for each satellite and writing Newton's Second Law gives $F_{net} = ma \rightarrow \frac{GmM}{r^2} = ma \rightarrow a = \frac{GM}{r^2}$. Since the radius of satellite 2 is twice as great, the acceleration is ¼ as large compared to satellite 1. As for the speed, we write $a = \frac{v^2}{r}$ and discover that $\frac{v^2}{r} = \frac{GM}{r^2} \rightarrow v = \sqrt{\frac{GM}{r}}$. Hence, satellite 2 will be slower by a factor of $\sqrt{2}$.

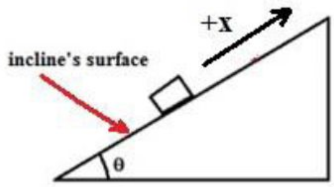
23. A car moves with constant speed around a horseshoe-shaped path as shown with the arrows in the figure. Which one of the following choices best describes the direction of the average acceleration of the car in traveling from W to X?



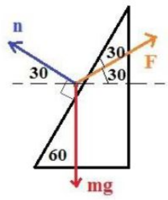
(A) ↙ **(B)** ↖ **(C)** ↗ **(D)** ↘ **(E)** There is no average acceleration

A… From the definition of average acceleration, we compute $\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t}$. Looking only at directions of vectors, we have $\langle\vec{a}\rangle = \frac{1}{\Delta t}(\vec{v}_f - \vec{v}_0) = \frac{1}{\Delta t}(\downarrow -(\rightarrow)) = \frac{1}{\Delta t}(\downarrow + \leftarrow) = \swarrow$.

24. A mass on a frictionless incline has a gravitational force, a normal force from the incline, and a force applied by a person that all are equal in magnitude. The mass remains at rest and the incline makes an angle θ counterclockwise from the horizontal. Which one of the following choices best describes the orientation of the applied force by the person? The +x-axis is directed upward, parallel to the incline's surface as shown in the figure.



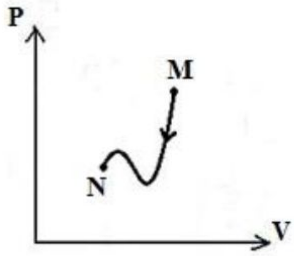
(A) The applied force is oriented directly along the +x axis.
(B) The applied force is oriented at an angle θ clockwise from the +x axis.
(C) The applied force is oriented at an angle $90^\circ - \theta$ clockwise from the +x axis.
(D) The applied force is oriented at an angle $90^\circ - \theta$ counterclockwise from the +x axis.
(E) This is a completely impossible situation that never can be realized physically.



C… In order for the sum of 3 equal-sized vectors to be zero, the angle between each vector with any other needs to 120°. Since the gravitational force is straight downward, the normal force is directed 30° above the horizontal. A picture is useful! From the construction, we see that the angle $\theta = 60^\circ$ making the applied force half-way between the horizontal and the level of the incline. So the applied force would have to be 30° below the incline's surface. This matches answer C.

25. A gas undergoes the unusual process $M \to N$ in the pressure vs. volume graph shown. Which one of the following choices properly represents the signs of the internal energy change of the gas, ΔU, the total energy transferred as heat to the gas, Q, and the total work done on the gas by the surroundings, W, for this process?

	ΔU	Q	W
(A)	Positive	Negative	Positive
(B)	Negative	Positive	Negative
(C)	Negative	Negative	Negative
(D)	Positive	Positive	Negative
(E)	Negative	Negative	Positive



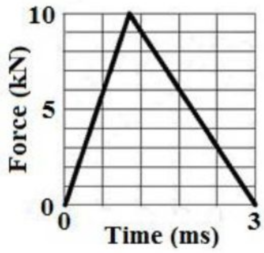
E… Since the pressure and volume decreased, from $PV = nRT$, the temperature must decrease. This means that the internal energy change is negative since the internal energy depends on the temperature for an ideal gas. Since the volume decreases, the work done on the gas by the surroundings is positive (force is directed inward on gas and movable piston of container moves inward). Finally, from the First Law of Thermodynamics, $\Delta U = Q + W \Rightarrow (-) = Q + (+)$. This means that the quantity of heat in this process will have to be negative $Q = (-) + (-) = (-)$.

26. The position of a mass connected to a spring obeys $x(t) = A\cos(\omega t)$. What is the average speed of the mass for one full oscillation in terms of the mass's maximum speed during oscillation, v_{max}?

(A) $\frac{2}{\pi}\, v_{max}$ **(B)** $\frac{1}{\sqrt{2}}\, v_{max}$ **(C)** $\frac{1}{2}\, v_{max}$ **(D)** $\frac{\sqrt{2}}{\pi}\, v_{max}$ **(E)** $\frac{1}{2\pi\sqrt{2}}\, v_{max}$

A… The maximum speed during oscillation is $v_{max} = \omega A = \frac{2\pi A}{T}$ which could be found in multiple ways, including equating the maximum KE during oscillation to the maximum potential energy $\left(\frac{1}{2}kA^2 = \frac{1}{2}mv_m^2 \to v_m = \sqrt{\frac{k}{m}}\sqrt{A^2} = \omega A\right)$. Now, for one full oscillation, the mass moves from full extension to equilibrium (A), then to full compression (A) and back again (2A) for a total distance traveled of 4A. Consequently, $v_{avg} = \frac{d}{t} = \frac{4A}{T} = 4\left(\frac{A\omega}{2\pi}\right) = 4\left(\frac{v_{max}}{2\pi}\right) = \frac{2}{\pi}\, v_{max}$

Questions 27 – 28 deal with the following information:
On a frictionless horizontal surface, two bodies make a head-on collision and stick together. Body 1 has a mass of $3.50\ kg$ and initially moves to the right with speed $7.0\frac{m}{s}$. Body 2 initially is at rest. A graph of the force exerted onto Body 2 from Body 1 during the collision is shown.



27. What is the mass of Body 2?

(A) $2.81\ kg$ **(B)** $3.50\ kg$ **(C)** $4.59\ kg$ **(D)** $5.53\ kg$ **(E)** $7.50\ kg$

D… The area under the force-time curve gives the impulse on mass 2. By Newton's Third Law, the force on mass 1 has the same magnitude but is in the opposite direction. The area is computed as $\frac{1}{2}(10kN)(3ms) = 15\ Ns$. So, the impulse for mass 1 is $-15\ Ns$. Using the impulse-momentum theorem, $\Delta p_1 = -15 = m_1(v_{1f} - v_{1i}) \Rightarrow -\frac{15}{3.50} = v_f - 7.0 \Rightarrow v_f = 2.71\frac{m}{s}$. So, using the impulse on mass 2, we find $\Delta p_2 = 15 = m_2(v_{2f} - v_{2i}) \Rightarrow m_2 = \frac{15}{2.71-0} = 5.54\ kg$.

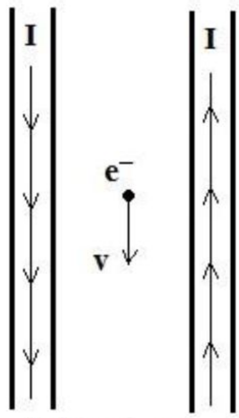
28. How much kinetic energy was transformed to other kinds of energy from the collision?

(A) $67.1\ J$ **(B)** $52.6\ J$ **(C)** $42.9\ J$ **(D)** $38.2\ J$ **(E)** $30.0\ J$

B… The initial kinetic energy of the system is $\frac{1}{2}m_1v_1^2 = \frac{1}{2}(3.50)(7.0)^2 = 85.75\ J$. After the collision, the kinetic energy is computed as $\frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(3.50 + 5.53)(2.71)^2 = 33.16\ J$. The difference in these quantities is $\Delta KE = KE_f - KE_i = 33.16 - 85.75 = -52.6\ J$.

29. An electron moves at constant non-zero velocity directly between two long straight wires. The conventional current in each wire has the same magnitude, but the currents are in opposite directions as shown in the figure. Ignoring gravity, which choice best reflects the direction of the magnetic field and the direction of the electric field that exist at the location of the electron? Any electric field in the region originates from an unseen external source.

	Electric Field	**Magnetic Field**
(A)	No field	No field
(B)	To the left	Into the plane of the page
(C)	To the right	Into the plane of the page
(D)	To the left	Out of the plane of the page
(E)	To the right	Out of the plane of the page



E… By using the right hand rule with the thumb directed along the current, we find that the wire on the right produces a field directed out of the plane of the page at the electron's location. Performing the same procedure for the left wire, that field also is directed out of the plane of the page. So, the total field is out of the plane of the page and from $\vec{F} = q\vec{v} \times \vec{B}$, the magnetic force is directed to the right (the cross term is to the left, but the electron makes the force to the right). Hence, we now need an electric force directed to the left to balance the magnetic force. From $\vec{F} = q\vec{E} \rightarrow \vec{E} = \frac{\vec{F}}{q}$, we see that if the force is to the left, then the field must be to the right since an electron is a negative charge.

30. A spring scale reads 2.50 N when a small solid mass hangs from it in air. The spring scale reads 1.58 N when the mass at the end of the spring is completely submerged in a container of water. Which one of the following choices best represents the density of the solid mass?

(A) $3.68 \times 10^3 \frac{kg}{m^3}$ **(B)** $2.72 \times 10^3 \frac{kg}{m^3}$ **(C)** $1.58 \times 10^3 \frac{kg}{m^3}$ **(D)** $9.20 \times 10^2 \frac{kg}{m^3}$ **(E)** $1.58 \times 10^2 \frac{kg}{m^3}$

B… The free body diagram of the solid mass in the water has three forces acting … gravitational, a spring force from the scale, and a buoyant force from the water. Writing Newton's Second Law, we have $F_{net} = ma \rightarrow B + T - mg = 0$. We have from the measurement in the air (since the buoyant force on the small mass from the air will be effectively negligible), that $mg = 2.50\ N$. Also, we have that $T = 1.58\ N$. Solving for the buoyant force yields $B = 2.50 - 1.58 = 0.92\ N$. The buoyant force is computed as $\rho_{water} g\ V = 0.92 \Rightarrow V = \frac{0.92}{(1000)(10)} = 9.2 \times 10^{-5} m^3$. The mass of the object is found as $m = \frac{2.50}{10} = 0.250\ kg$ leading to a density of the mass as $\rho = \frac{m}{V} = \frac{0.250}{9.2\times10^{-5}} = 2.7 \times 10^3 \frac{kg}{m^3}$

31. Approximately how many hydrogen atoms are there in the liquid water of Earth's oceans?

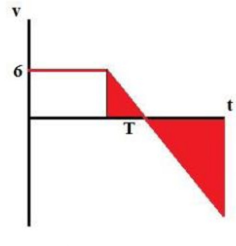
(A) 10^{62} **(B)** 10^{57} **(C)** 10^{52} **(D)** 10^{47} **(E)** 10^{42}

D… Approximately 70% of the Earth's surface is water. Approximating the volume of water with the surface area of the earth multiplied by a depth of about 3 mile (5000 meters approximately), we have $V_{water} = \frac{7}{10}\ 4\pi r^2(3\ mi) = \frac{28}{10}\pi(6.4 \times 10^6)^2(5000) = 1.8 \times 10^{18} m^3$. The density of water is $1000 \frac{kg}{m^3}$ and so the total mass of water in the oceans is $1.8 \times 10^{18} * 1000 = 1.8 \times 10^{21} kg$. The molar mass of water is $18 \frac{g}{mol}$ $(H = 1, O = 16)$ yielding $n = \frac{1.8\times10^{21}}{0.018} = 1.0 \times 10^{23}\ mol$. There are 2 hydrogen atoms per water molecule, and so we need the number of molecules from Avogadro… $N = nN_A = (1.0 \times 10^{23})(6.02 \times 10^{23}) = 6.0 \times 10^{46}$ giving $2(6.0 \times 10^{46}) = 1.2 \times 10^{47}$ H atoms.
*** http://water.usgs.gov/edu/earthwherewater.html is where this question was vetted.

32. An object moving along a line completes a 20.0 second trip with an average speed of 10.0 $m/_s$ in two stages. During stage 1, the object moves with a constant velocity of 6.0 $m/_s$ to the right for 12.0 seconds. What constant magnitude acceleration directed to the left must the object have during the 8.0 seconds of stage 2?

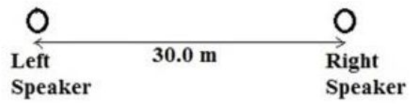
(A) 2.5 $m/_{s^2}$ **(B)** 2.7 $m/_{s^2}$ **(C)** 4.0 $m/_{s^2}$ **(D)** 5.3 $m/_{s^2}$ **(E)** 6.3 $m/_{s^2}$

D… For the average speed to be $10\frac{m}{s}$, a total distance of 200 m must be traveled. In the first 12 seconds, a total of $\Delta x = \langle v \rangle \Delta t = \left(6.0\frac{m}{s}\right)(12s) = 72\ m$ has been traveled. This means that in the last 8 seconds, a distance of 128 m needs to be traversed. Let's draw a graph of velocity vs time. The shaded regions have to add to a total distance of 128 m. In the first T seconds, the object slows to rest. Mathematically, $|a| = \frac{6}{T}$ with area $\frac{1}{2}(6)(T) = 3T$. For the remaining time, the area is $\frac{1}{2}(8 - T)(a(8 - T)) = \frac{1}{2}a(8 - T)^2$. Hence, $3T + \frac{1}{2}a(8 - T)^2 = 128$.



Substituting the expression for the acceleration and multiplying by T yields $3T^2 + 3(8 - T)^2 = 128\,T \Rightarrow 3T^2 - 88T + 96 = 0$. Solving this expression yields times of $T = 1.13\ s, 28.2\ s$. Using $T = 1.13\ s$ gives $a = \frac{6}{1.13} = 5.3\frac{m}{s^2}$.

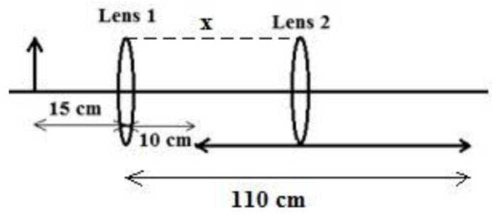
33. Two spherical speakers separated by 30.0 m each emit a constant frequency signal of 57.0 Hz in phase with each other. The speed of sound is 342 $\frac{m}{s}$. How many locations of complete destructive interference of the incoming signals are there on the line between the speakers?



(A) 12 **(B)** 11 **(C)** 10 **(D)** 9 **(E)** 6

C… The wavelength of the sounds is $v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{342}{57} = 6.0\ m$. This means that the speakers are separated by exactly 5 wavelengths. For constructive interference, the path difference between the waves from the speakers must differ than an integer number of wavelengths. Hence, there are 11 locations at or between the speakers where this happens $-5\lambda, -4\lambda, -3\lambda, \ldots 3\lambda, 4\lambda, 5\lambda$. A place of destructive interference occurs directly between the constructive interference locations… hence, there are 10 of them.

34. An upward-pointing object is placed 15.0 cm to the left of a lens system. The first lens is convex with focal length 10.0 cm. The second lens is convex with focal length 10 cm and its location from the first lens is varied from 10 cm away to 110 cm away. Which one of the following choices best represents the description of the final image formed as the second lens is moved from $x = 10\ cm$ to $x = 110\ cm$ from the first lens?



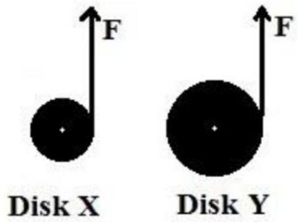
	$x = 10\ cm\ away$ → → → → → → →		→ $x = 110\ cm\ away$
(A)	Real & pointing downward	Virtual & pointing downward	Real & pointing upward
(B)	Virtual & pointing downward	Virtual & pointing upward	Real & pointing upward
(C)	Virtual & pointing upward	Virtual & pointing downward	Real & pointing downward
(D)	Real & pointing upward	Virtual & pointing downward	Real & pointing upward
(E)	Virtual & pointing downward		Real & pointing upward

A… The image from the first lens is found from $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \to \frac{1}{10} = \frac{1}{15} + \frac{1}{q} \to q = 30\ cm$.

- **Far from 1st lens** (more than 40 cm from the first lens) - we have that the image from the first lens is the object for the second lens and that the object is greater than a focal length from the 2nd lens. This means that we will end up with a real image from the 2nd lens and the image will be flipped from the "object". Since the 1st lens formed a real image, that image was flipped from the original object. In other words, for large distance between the lenses, the image is *real and pointing upward.*
- **Medium range from 1st lens (**between 30 and 40 cm from the 1st lens) - the image from the first lens is now inside the focal length of the 2nd lens. Hence, between 30 and 40 cm, the image formed is *virtual and pointing downward* (there is still the inversion from the 1st lens).
- **Close range –** (less than 30 cm from the first lens) - we have a virtual object! For simplicity, let the 2nd lens be placed 20 cm from the first lens making $p = -10\ cm$ for the second lens. This gives $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \to \frac{1}{10} = \frac{1}{-10} + \frac{1}{q} \to q = 5\ cm$. Because we have a positive image position, the image is real. Further, the magnification of the image from the 2nd lens is $M = -\frac{q}{p} = -\left(\frac{5}{-10}\right) = \frac{1}{2}$. Because it is positive, the image has the same orientation as the object. The object for this lens was the upside-down image from the first lens. Hence, the image is *real and pointing downward.*

35. Two uniform disks, X and Y, have equal masses, M, but different radii such that $r_X < r_Y$. Both disks initially are at rest. A force F is applied tangent to each disk at its right edge for the same amount of time. As a result, each disk rotates counterclockwise in the plane of the page about a fixed frictionless axis through its center. Which one of the following choices correctly compares the magnitudes of angular momentum L about the center axis and total kinetic energy K of disk X and disk Y?

(A) $L_X < L_Y$; $K_X < K_Y$
(B) $L_X < L_Y$; $K_X > K_Y$
(C) $L_X = L_Y$; $K_X = K_Y$
(D) $L_X = L_Y$; $K_X < K_Y$
(E) $L_X < L_Y$; $K_X = K_Y$



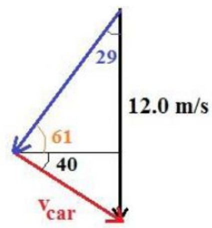
E … From the angular impulse-momentum theorem, the change in angular momentum is the torque multiplied by the time. As the forces are equal but at different distances from the center of each object, $\tau_X < \tau_Y$ meaning that $L_X < L_Y$ since the forces are applied for the same time. As for the kinetic energy, we note that $KE = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} = \frac{\tau^2 t^2}{2\left(\frac{1}{2}Mr^2\right)} = \frac{r^2F^2t^2}{Mr^2} = \frac{F^2t^2}{M}$. All of these quantities (force, time, mass) were equal for the two disks thereby making the kinetic energy the same for the disks!

36. Rain falls vertically at 12.0 m/s with respect to a stationary observer. A car is moving at an angle of 40° below the horizontal with respect to the observer. A passenger sitting in the car notes that the rain makes an angle of 29.0° with the vertical. What is the car's speed with respect to the observer?

(A) $2.29\frac{m}{s}$ **(B)** $5.93\frac{m}{s}$ **(C)** $9.03\frac{m}{s}$ **(D)** $11.8\frac{m}{s}$ **(E)** $16.2\frac{m}{s}$

B… METHOD #1: algebra - One way to handle this problem is to make little pictures and use subscripts. We write $\vec{v}_{rain\,wrt\,Earth} = (0)\hat{x} + (-12)\hat{y}$, $\vec{v}_{car\,wrt\,Earth} = (v_{car}\cos 40)\hat{x} + (-v_{car}\sin 40)\hat{y} = (0.776\, v_{car})\hat{x} + (-0.643\, v_{car})\hat{y}$. Now, to find the velocity of the rain with respect to the car, we compute $\vec{v}_{rain\,wrt\,car} + \vec{v}_{car\,wrt\,Earth} = \vec{v}_{rain\,wrt\,Earth}$. Hence, we rearrange this expression to find $\vec{v}_{rain\,wrt\,car} = \vec{v}_{rain\,wrt\,Earth} - \vec{v}_{car\,wrt\,Earth} = (0 - 0.776\, v_{car})\hat{x} + (-12 + 0.643\, v_{car})\hat{y}$. Since these components of the velocity are known to make a 29 degree angle with the vertical, we can write $\tan 29^0 = \frac{(-0.776\, v_{car})}{-12+0.643 v_{car}}$. Using a little algebra, we can solve this equation for the unknown v_{car}. Doing this leads to $v_{car} = 5.93\frac{m}{s}$.

METHOD #2: pictorial – Using $\vec{v}_{rain\,wrt\,car} + \vec{v}_{car\,wrt\,Earth} = \vec{v}_{rain\,wrt\,Earth}$, we construct the picture shown to the right for the velocities. Hence, from the Law of Sines, $\frac{12}{\sin 101} = \frac{v_{car}}{\sin 29} \Rightarrow v_{car} = \frac{12 \sin 29^\circ}{\sin 101^\circ} = 5.93\frac{m}{s}$



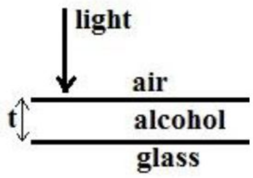
37. Which one of the following choices represents the base SI units of inductance?

(A) $\frac{kg\, m^2}{A^2\, s^2}$ **(B)** $\frac{kg\, m^2}{A\, s}$ **(C)** $\frac{kg\, m}{A^2\, s^2}$ **(D)** $\frac{kg\, m^2}{A^2\, s^3}$ **(E)** $\frac{kg\, m}{A^2\, s^3}$

A… The energy stored by an inductor takes the form $U = \frac{1}{2}LI^2$ and be rearranging, the units of inductance are found from

$$\frac{U}{I^2} \to \frac{J}{A^2} = \frac{Nm}{A^2} = \frac{\left(kg\frac{m}{s^2}\right)m}{A^2} = \frac{kg\, m^2}{A^2 s^2}$$

38. A thin film of alcohol ($n_{alcohol} = 1.35$) lies on a flat glass surface ($n_{glass} = 1.60$). When light of wavelength 540 nm is incident normal to the alcohol surface from air, the light is strongly reflected, but when light of wavelength 432 nm is incident normal to the surface from air, the reflected light is minimized. Which one of the following choices could represent the thickness, t, of the alcohol film?

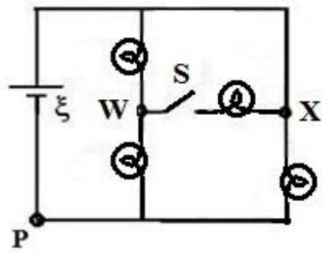


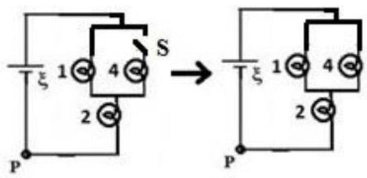
(A) 216 nm **(B)** 320 nm **(C)** 324 nm **(D)** 400 nm **(E)** 486 nm

D… To create constructive interference with the 540 nm light, we need to be sure that the interfering rays from the reflection off the alcohol and the ray that passes into the alcohol and is reflected back at the glass surface are in phase. Since the light is traveling from air to alcohol to glass, the index of refraction increases at each interface, meaning that the reflected light is phase-shifted by $\frac{\lambda}{2}$. This means that for light traveling down through the alcohol a distance t to the glass surface and then traveling an additional distance t back to the air, we need this extra path length to be an integer number of wavelengths to put the our two waves in phase. That is, $2t = m\frac{\lambda}{n}$. Solving for $t = \frac{m\lambda}{2n} = m\left(\frac{540}{(2)(1.35)}\right) = 200\ m$. Since m is an integer, the only possible choice would be $t = 400\ nm$. Likewise, destructive interference is needed for the 432 nm light and that condition is$2t = \left(p + \frac{1}{2}\right)\frac{\lambda}{n}$. Solving this expression with $p = 2$ also yields that $t = 400\ nm$.

39. For the circuit shown, the four light bulbs have identical resistance, the battery is ideal and all wires have no resistance. After the switch, S, in the circuit is closed, which one of the following choices correctly describes what happens to the magnitude of the current at the point labeled P and to the magnitude of the potential difference from W to X?

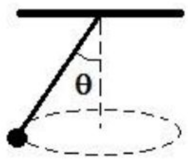
	Current at P	ΔV_{XW}
(A)	No change	Increases
(B)	Decreases	Increases
(C)	Increases	Increases
(D)	Decreases	Decreases
(E)	Increases	Decreases





E… By use a Kirchhoff loop with the battery and bulb 3, we see that there is no difference in the voltage or current through it whether the switch is open or closed. By looking at what is left, from Kirchhoff's Loop Rule, the potential difference across bulb 1 is $^{\xi}/_{2}$ with the switch open. After closing the switch, the effective resistance of the bulb 1-4 branch decreases resulting in more current through bulb 2 (and P). This increases the potential difference across bulb 2 thereby decreasing the potential difference across bulb 1 and the branch from W to X!

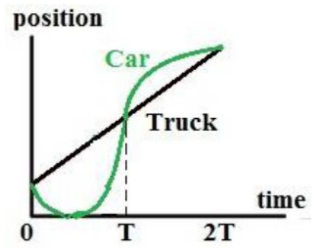
40. A 2.0 kg mass is connected to the end of string and moves about the string's fixed end in a conical motion with a constant speed of $4.0\frac{m}{s}$. The string has a length of 2.50 m and forms an angle of θ with the vertical. What is the tension in the string?



(A) 20.0 N **(B)** 23.7 N **(C)** 27.4 N **(D)** 29.8 N **(E)** 32.5 N

C… From the free body diagram of the mass, there are two forces: gravitational and tension. Writing Newton's Second Law for each component of motion, we have $F_{net_y} = ma_y \rightarrow T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg$ and $F_{net_x} = ma_x \rightarrow T\sin\theta = m\left(\frac{v^2}{r}\right) = m\left(\frac{v^2}{L\sin\theta}\right) \Rightarrow T\sin^2\theta = \frac{mv^2}{L}$. To find the tension directly, we'll take the y-component expression and square it. After this, we multiply the x-component by T leading to $T^2\sin^2\theta = \frac{mv^2}{L}T$ and $T^2\cos^2\theta = (mg)^2$. Adding these relations (note that $\sin^2\theta + \cos^2\theta = 1$) gives $T^2 - \left(\frac{mv^2}{L}\right)T - (mg)^2 = 0 \Rightarrow T^2 - 12.8\,T - 400 = 0$. Using the Pythagorean Theorem yields $T = 27.4\ N;\ -14.6\ N$. The physical answer therefore is $T = 27.4\ N$.

41. For the entire time shown in the graph, which one of the following choices correctly describes the relationship between the average speed of the truck to that of the car?



(A) The truck's average speed is less than the average speed of the car.
(B) The truck's average speed is the same as the average speed of the car.
(C) The truck's average speed is greater than the average speed of the car.
(D) The truck's average speed is positive while the car's average speed is negative but of the same magnitude.
(E) A relationship cannot be determined without more information.

A… The average speed is distance divided by time. The truck moves at constant rate from the start to end position while the car initially moves "backward" and then forward to the same ending location of the truck. Consequently, the car travels a greater distance than the truck and has a higher average speed.

42. Which one of the following choices best describes the instants of time, t, at which the car and truck travel with the same speed?

(A) Only at times $t = 0, t = T$ and $t = 2T$.
(B) At one instant during the interval $0 < t < T$ and at one instant during the interval $T < t < 2T$.
(C) At two instants during the interval $0 < t < T$ and at one instant during the interval $T < t < 2T$.
(D) At one instant during the interval $0 < t < T$ and at two instants during the interval $T < t < 2T$.
(E) At two instants during the interval $0 < t < T$ and at two instants during the interval $T < t < 2T$.

C… The slope of the position vs, time graph. The car moves faster than the truck initially and eventually come to rest, which means that at some point, the speed of the car and truck are the same. From rest, the car accelerates and catches up to the truck by eventually moving faster. This means that the car and truck have the same speed again before time T. Finally, for times $T < t < 2T$, the car slows down close to rest and the truck catches up. Hence, the car's speed is again equal to that of the truck at some time. This makes C the correct answer.

43. An object of mass 4.0 kg has a total kinetic energy of 100.0 J and an x-component of linear momentum equal to 24.0 $kg\frac{m}{s}$. The object is moving in the x-y plane. What is the y-component of the object's linear momentum?

(A) $8.00\ kg\frac{m}{s}$ **(B)** $15.0\ kg\frac{m}{s}$ **(C)** $26.0\ kg\frac{m}{s}$ **(D)** $32.0\ kg\frac{m}{s}$ **(E)** $97.0\ kg\frac{m}{s}$

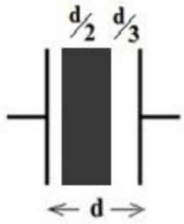
B… Since $K = \frac{p^2}{2m}$, we can write that $K = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} \Rightarrow 100 = \frac{(24)^2}{2(4)} + \frac{p_y^2}{2(4)} \Rightarrow 800 = 576 + p_y^2 \Rightarrow p_y^2 = 224$. Hence, $p_y = \sqrt{224} = 14.97\ kg\frac{m}{s} = 15.0\ kg\frac{m}{s}$

44. Which one of the following choices is most associated with the following statement: "When the pressure of a gas is held constant, the volume of the gas is directly proportional to the temperature." ?

(A) Newton's Law **(B)** Boyle's Law **(C)** Avogadro's Law **(D)** Graham's Law **(E)** Charles's Law

E… Jacques Charles is the scientist associated with the observation that volume is proportional to temperature at constant pressure.

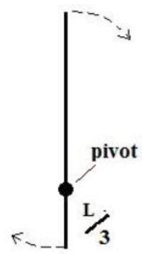
45. A 6.00 μF parallel-plate capacitor is disconnected from a 12 volt battery after being fully charged. A person now carefully inserts a dielectric material of constant $\kappa = 2$ so that it fills one-half of the space between the plates as shown. How much work was done by the person while inserting the dielectric?



(A) $-81\ \mu J$ **(B)** $-108\ \mu J$ **(C)** $-144\ \mu J$ **(D)** $-216\ \mu J$ **(E)** $-288\ \mu J$

B… The work done by the person inserting the dielectric would be equal to the change in the potential energy for the capacitor. Since the battery was disconnected, the charge on the plates remains constant and $W_{person} = \Delta U = \frac{1}{2} Q\Delta V$. The charge on the plates would be $Q = CV = (6.00\ \mu F)(12.0\ v) = 72.0\ \mu C$. The final capacitance can be found by moving the dielectric to any location in the space between plates and treating the system as 2 capacitors in series. The potential difference through the air-filled portion would be 6 volts since the field strength is unchanged and we are crossing half the original capacitor (which had 12 volts). For the dielectric portion, the dielectric decreases the field strength by a factor of 2… which means that the potential difference across the dielectric would be 3 volts for a grand total of 9 volts across the entire capacitor. Hence, $W_{person} = \frac{1}{2} Q\Delta V = \frac{1}{2}(72\mu C)(-3v) = -108\ \mu J$.

46. A uniform rod of mass M and length L is fixed to rotate about a frictionless pivot located $L/_3$ from one end. The rod is released from rest incrementally away from being perfectly vertical, resulting in the rod rotating clockwise about the pivot. When the rod is horizontal, what is the magnitude of the tangential acceleration of its center of mass?



(A) $\frac{1}{6} g$ **(B)** $\frac{1}{2} g$ **(C)** $\frac{4}{3} g$ **(D)** $\frac{2}{3} g$ **(E)** $\frac{1}{4} g$

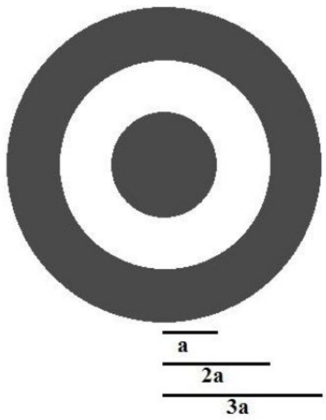
E… We write $\tau_{net} = I\alpha$ and note that the moment of inertia of the rod is $I = \frac{1}{12} ML^2$ about an axis through the center of mass. To find the moment of inertia about the pivot, we need the parallel axis theorem to obtain $I = \frac{1}{12} ML^2 + md^2$ where $d = \frac{L}{6}$ (the distance between the center of the mass and the pivot) leading to $I = \frac{1}{12} ML^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{9} ML^2$. Calculating the torque from the gravitational force at the center of the stick gives $\tau = Fd\sin\theta = Mg\left(\frac{L}{6}\right)\sin -90^\circ = -\frac{MgL}{6}$. (The minus sign for clockwise). So, $\tau_{net} = I\alpha \Rightarrow -\frac{MgL}{6} = \frac{1}{9} ML^2\alpha \Rightarrow \alpha = -\frac{3}{2}\frac{g}{L}$. Using $a_t = r\alpha$ leads to $a_t = \left(\frac{L}{6}\right)\left(-\frac{3}{2}\frac{g}{L}\right) \Rightarrow |a_t| = \frac{g}{4}$.

47. One mole of a diatomic ideal gas undergoes a reversible adiabatic process. The pressure and volume initially are given as $P = 2.0\ atm$ and $V = 30\ L$. If the volume is halved during the adiabatic process, how much work was done on the gas sample by the surroundings?

(A) $6790\ J$ **(B)** $5530\ J$ **(C)** $4850\ J$ **(D)** $4200\ J$ **(E)** $3040\ J$

C… From the First Law of Thermodynamics, $\Delta U = Q + W$, and since it is an adiabatic process $Q = 0$. We can therefore find the work done by looking at the internal energy change with $\Delta U = n\left(\frac{5}{2}R\right)\Delta T$ for a diatomic gas. Using $PV = nRT \Rightarrow T = \frac{PV}{nR} = \frac{(2.0\ atm)(30\ L)}{(1\ mol)\left(0.0821\ L\cdot\frac{atm}{mol}\cdot K\right)} = 730.8\ K$. For an adiabat, $PV^\gamma = constant$ and $P = \frac{nRT}{V}$ which upon substitution gives $TV^{\gamma-1} = Const$. Hence, $(730.8)(30)^{\frac{2}{5}} = T_f(15)^{\frac{2}{5}} \Rightarrow T_f = 964.3\ K$. Finally, $W = \Delta U = n\left(\frac{5}{2}R\right)(964.3 - 730.8) = 4850\ J$

48. Two concentric charged conducting shells are in free space. The outer shell has inner radius $2a$ and outer radius $3a$. The inner shell has radius a. It is known that the electric potential at $r = 3a$ is $V_{3a} = {}^{kQ}/_{3a}$. If the electric potential V_a at $r = a$ is 0 volts, what is the charge on the inner spherical shell, Q_{in}?



(A) $Q_{in} = -{}^{3}/_{2}\, Q$
(B) $Q_{in} = -{}^{2}/_{3}\, Q$
(C) $Q_{in} = -{}^{1}/_{3}\, Q$
(D) $Q_{in} = -2Q$
(E) $Q_{in} = -{}^{1}/_{2}\, Q$

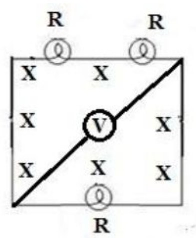
B… Outside the shell of radius $3a$, the total field will be that of a point charge and so will the potential giving $V_{3a} = \frac{kQ}{3a}$. Between 2a and 3a, there is no field interior to the conductor (since the shells are in free space and there is no mention of any other charges nearby… the system quickly attains static equilibrium) and so the potential at $2a$ is given as $V_{2a} = \frac{kQ}{3a}$. Noting that there is a field between a and 2a, we have a potential difference computed as $\Delta V = V_f - V_i$ where $V_f = V_a$ and $V_i = V_{2a}$ and on the left side we write $\Delta V = V(r) - V(2a) = \frac{kQ_{in}}{a} - \frac{kQ_{in}}{2a}$. And so, we have $\frac{kQ_{in}}{a} - \frac{kQ_{in}}{2a} = 0 - \frac{kQ}{3a}$. This leads to $\frac{kQ_{in}}{2a} = -\frac{kQ}{3a} \Rightarrow Q_{in} = -\frac{2}{3}Q$

49. The kinetic energy associated with an electron is twice its rest energy. At what speed is the electron traveling?

(A) $2.83 \times 10^8 \frac{m}{s}$ **(B)** $2.67 \times 10^8 \frac{m}{s}$ **(C)** $2.60 \times 10^8 \frac{m}{s}$ **(D)** $2.25 \times 10^8 \frac{m}{s}$ **(E)** $2.12 \times 10^8 \frac{m}{s}$

A… From relativity, we know $KE = (\gamma - 1)m_0c^2$ and so, $2m_0c^2 = (\gamma - 1)m_0c^2 \Rightarrow \gamma = 3$. From this, $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 3 \Rightarrow \frac{1}{9} = 1 - \frac{v^2}{c^2} \Rightarrow v^2 = \frac{8}{9}c^2 \Rightarrow v = \sqrt{\frac{8}{9}}c = 2.83 \times 10^8 \frac{m}{s}$

50. A magnetic field directed into the plane of the page is decreasing in time. A constant emf ξ is produced for the square loop enclosing the field in the figure. The square loop has three identical light bulbs of resistance R in it and an ideal voltmeter connected to the corners through the center of the loop. What is the magnitude of the voltmeter's reading?



(A) $0\,\xi$ **(B)** $\frac{1}{2}\xi$ **(C)** $\frac{1}{3}\xi$ **(D)** $\frac{1}{6}\xi$ **(E)** $\frac{2}{3}\xi$

D… By traversing a loop around the outside, we enclose an entire emf ξ which would be equally distributed across each identical resistor since the outer loop is the only one with current. Hence, the voltage is $\frac{\xi}{3}$ for any resistor. By traversing the top triangular loop, we enclose $\frac{\xi}{2}$ and have 2 resistors dropping potential $\frac{\xi}{3}$, we have $\frac{\xi}{2} = 2\left(\frac{\xi}{3}\right) + V_{voltmeter} \Rightarrow V_{voltmeter} = -\frac{\xi}{6}$. As a check, the lower triangular loop gives $\frac{\xi}{2} = \left(\frac{\xi}{3}\right) - V_{voltmeter} \Rightarrow V_{voltmeter} = -\frac{\xi}{6}$. The magnitude of the voltage is therefore $\frac{\xi}{6}$.